ASTAM Fall 2023 Model Solutions

Revised February 20, 2024

ASTAM Fall 2023 Q1

(a) (i)

$$E[S] = \lambda E[X] = 1.9\left(\frac{900}{3}\right) = 570$$

$$V[S] = V[X]E[N] + V[N](E[X])^2 = \left(\frac{900}{3}\right)^2\left(\frac{4}{2}\right)(1.9) + (1.9)(300)^2 = 513{,}000 = 716.24^2$$

Or because it is Poisson:

$$V[S] = \lambda E\left[X^2\right] = 1.9\left(\frac{2\times 900^2}{3\times 2}\right) = 513{,}000 = 716.24^2$$

(b) With 20% inflation, the distribution of individual claims for 2024 will be adjusted by a factor of 1.2. Because Pareto is a scale family with scale parameter θ, the distribution of individual claims in 2024 will also have a Pareto with parameters $\alpha = 4$ and adjusted scale parameter $\theta = 1080$.

(c) (i)

$$X \sim \text{Pareto}(\alpha = 4,\ \theta^* = 1080)$$

$$\lambda E\left[(X-d)_+\right] = 570 \Rightarrow E\left[(X-d)_+\right] = \frac{570}{1.9} = 300$$

$$E\left[(X-d)_+\right] = \Pr\left[X > d\right] E\left[\,X - d \middle| X > d\right]$$

$$= \left(\frac{\theta^*}{\theta^* + d}\right)^4 \frac{\theta^* + d}{3} \qquad \text{(formula sheet)}$$

$$\Rightarrow \left(\frac{(\theta^*)^4}{\left(\theta^* + d\right)^3}\right) = 900 \qquad \text{where } \theta^* = 1080$$

$$\Rightarrow \theta^* + d = 1147.7 \Rightarrow d = 67.67$$

Or

$$300 = E\left[(X-d)_+\right] = E[X] - E\left[X \wedge d\right]$$

$$= \frac{\theta^*}{3} - \frac{\theta^*}{3}\left(1 - \left(\frac{\theta^*}{\theta^* + d}\right)^3\right) \qquad \text{(formula sheet)}$$

$$= \left(\frac{(\theta^*)^4}{3\left(\theta^* + d\right)^3}\right)$$

(ii)

$$N^* \sim \text{Poisson}\left(\lambda \Pr\left[X > d\right]\right)$$

$$E\left[N^*\right] = 1.9\left(\frac{1080}{1080 + 67.67}\right)^4 = 1.4898$$

(iii) The deductible might affect policyholder behavior.

- Policyholders may not wish to go through the trouble of making a claim for amounts just slightly above the deductible.
- Policyholders may go to the dentist less, now that each visit costs them the deductible. That could mean that each visit is more costly, on average.
- Policyholder may discontinue policy or look for other policies in the market.

(d)

Now $E[X \wedge L] - E[X \wedge 50] = 300$

$$\Rightarrow \frac{1080}{3}\left(1-\left(\frac{1080}{1080+L}\right)^3\right) - \frac{1080}{3}\left(1-\left(\frac{1080}{1080+50}\right)^3\right) = 300$$

$$\Rightarrow \left(\frac{1080}{1130}\right)^3 - \left(\frac{1080}{1080+L}\right)^3 = \frac{900}{1080}$$

$$\Rightarrow 1080 + L = 3165.6$$

$$\Rightarrow L = 2085.6$$

Notes: The maximum insured loss using this notation is L-50=2035.6. Either answer was given full credit.

(e) Discretizing the 2023 severity distribution with $h = 100$, the probability is

$$f_0 = \Pr[X \leq 50] = 1 - \left(\frac{900}{950}\right)^4 = 0.19448$$

(f) For the discretized loss distribution we have

$$\Pr[S=0] = \Pr[N=0] + \Pr[N=1] f_0 + \Pr[N=2] f_0^2 + \ldots$$

$$= E_N[f_0^N] = P_N(f_0) \qquad (P_N \text{ is the p.g.f})$$

$$= e^{\lambda(f_0 - 1)} = 0.21643$$

Or

$$f_S(0) = Pr[N=0] + Pr[N=1]\Pr[X=0] + Pr[N=2](\Pr[X=0])^2 + Pr[N=3](\Pr[X=0])^3 + \ldots$$

$$= e^{-1.9} + \frac{e^{-1.9}(1.9)}{1!}(0.19448) + \frac{e^{-1.9}(1.9)^2}{2!}(0.19448)^2 + \frac{e^{-1.9}(1.9)^3}{3!}(0.19448)^3 + \ldots$$

$$= 0.14957 + 0.05527 + 0.01021 + 0.00125 + 0.00012 + 0.00001 + 0 = 0.21643$$

Examiners' Comments

Overall, the candidates did very well on this question.

For Part a, virtually all candidates earned full credit.

For Part b, many candidates earned full credit by using the distribution function to prove the relationship or by pointing out that theta is a scale parameter so it is multiplied by 1.2. Full credit or no credit was given for other approaches or not even mentioning alpha. The question asks for an explanation. Quite a few candidates did not provide an explanation but merely repeated what was given.

Part c i, most candidates received full points. Proving that the deductible of 70 is very close to the correct deductible received minimal credit.

Part c ii, most candidates earned full credit.

Part c iii, many candidates treated the deductible as an annual deductible instead of a deductible per visit. Also, candidates did not discuss policyholder behavior but implications of adding a deductible in other ways. These candidates did not receive credit for either of these approaches.

Part d and e, this part was well answered by well-prepared candidates.

Part f, this part was not as well done as the other calculation questions.

ASTAM Fall 2023 Q2

(a)

The bodily injury category is more likely to have a heavy-tailed distribution than the vehicle repair category. Compensation for injuries is more likely to subject to litigation which often leads to claims in the right tail. On the other hand, the severity of vehicle repair is limited to the value of the vehicle.

(b)

(i) An increasing mean excess loss function indicates a heavy-tailed distribution.

(ii) An increasing hazard rate function indicates a light-tailed distribution.

For the Pareto distribution with $\alpha > 1$,

$$e_X(d) = E\left[X - d \middle| X > d\right]$$

$$= \frac{\int_d^\infty (x-d)\frac{\alpha\theta_X^\alpha}{\left(\theta_X + x\right)^{\alpha+1}}dx}{1 - F_X(d)}$$

$$= \left(\frac{\theta_X + d}{\theta_X}\right)^\alpha \left(\int_d^\infty (x-d)\frac{\alpha\theta_X^\alpha}{\left(\theta_X + x\right)^{\alpha+1}}dx\right)$$

Let $y = x - d$

$$e_X(d) = \left(\frac{\theta_X + d}{\theta_X}\right)^\alpha \left(\int_0^\infty y\frac{\alpha\theta_X^\alpha}{(\theta + d + y)^{\alpha+1}}dy\right)$$

$$= \left(\frac{\theta_X + d}{\theta_X}\right)^\alpha \left(\frac{\theta_X}{\theta_X + d}\right)^\alpha \left(\int_0^\infty y\frac{\alpha\left(\theta_X + d\right)^\alpha}{\left(\theta_X + d + y\right)^{\alpha+1}}dy\right)$$

$$= \int_0^\infty y\frac{\alpha\left(\theta_X + d\right)^\alpha}{\left(\theta_X + d + y\right)^{\alpha+1}}dy$$

Note that $\frac{\alpha\left(\theta_X + d\right)^\alpha}{\left(\theta_X + d + y\right)^{\alpha+1}}$ is the pdf of a Pareto$(\alpha, \theta_X + d)$ distribution,

so the integral is the mean of the Pareto distribution,

$$\Rightarrow e_X(d) = \frac{\theta_X + d}{\alpha - 1}$$

(ii) The hazard rate is given by

$$h(x) = \frac{f(x)}{S(x)} = \frac{\alpha\theta_X^\alpha}{\left(\theta_X + x\right)^{\alpha+1}} \left(\frac{\theta_X + x}{\theta_X}\right)^\alpha = \frac{\alpha}{\theta_X + x}$$

(c)

(i) For the Weibull distribution,

$$h(x) = \frac{f(x)}{S(x)} = \frac{f(x)}{1 - F(x)} = \frac{\frac{\tau x^{\tau-1}}{\theta^\tau} e^{-(x/\theta)^\tau}}{e^{-(x/\theta)^\tau}} = \frac{\tau x^{\tau-1}}{\theta^\tau}$$

(ii) We are looking for values of τ for which the hazard rate function is a decreasing function:

$$h(x) = \frac{\tau x^{\tau-1}}{\theta^\tau} \Rightarrow h'(x) = \frac{\tau(\tau-1)x^{\tau-2}}{\theta^\tau}$$

This derivative will be ≤ 0 for $0 \leq \tau \leq 1$ and hence these are the values for which the distribution has a heavy tail.

(d) The limit of the ratio of survival functions is:

$$\lim_{x\to\infty} \frac{S_X(x)}{S_Y(x)} = \lim_{x\to\infty} \frac{\theta_X^2}{\left(\theta_X + x\right)^2} e^{\left(x/\theta_Y\right)^{0.5}}$$

Let $y = \left(x/\theta_Y\right)^{0.5}$:

$$\lim_{x\to\infty} \frac{S_X(x)}{S_Y(x)} = \lim_{y\to\infty} \frac{\theta_X^2 e^y}{\left(\theta_X + \theta_Y y^2\right)^2} = \lim_{y\to\infty} \frac{\theta_X^2 e^y}{2\theta_X\theta_Y y^2 + \theta_X y^4}$$

$$= \lim_{y\to\infty} \frac{e^y}{y^2 + y^4} = \infty$$

Hence, *X* has a heavier tail than *Y*.

(e) As a distribution in the Gumbel MDA, *Y* must have an infinite number of moments as all positive moments exist.

Examiners' Comments

For Part (a) Most candidates correctly chose bodily injury, but some confused the fat tail of the loss distribution with the term 'long-tail' in explaining their reasoning.

Part (c) asked for a proof of the maximum excess loss formula for the Pareto distribution. For full credit candidates were expected to ***prove*** *the result starting from the definition of the function, but substantial partial credit was given to the candidates who used the excess loss formula from the formula sheet.*

For Part(e), the candidate needed to take the limit of the ratio between two survival functions. The ratio is an exponential function divided by polynomial as the argument goes to infinity. Full marks were given if the candidate demonstrated or even mentioned that an exponential function goes to infinity at a higher rate than a polynomial function. Using some arbitrary values to indicate the limit pattern was not acceptable.

ASTAM Fall 2023 Q3

(a) (i) The loglikelihood function is

$$l(\theta) = log\left(\prod_{i=1}^{n}\frac{1}{\theta}e^{-\frac{x_i}{\theta}}\right) = -n\ log\theta - \frac{1}{\theta}\sum_{i=1}^{n}x_i = -n\left(log\ \theta + \frac{\bar{x}}{\theta}\right)$$

The maximum is achieved when θ is the MLE, i.e. when $\hat{\theta}=\bar{x}=1015.92$, so

$$l_{max} = l(\hat{\theta}) = -1000(log\ \hat{\theta} + 1) = -7923.55$$

(ii) The K-S test statistic can be read off the graph as

$$D = \max_i |F_n(x_i) - F^*(x_i)| = 0.0275$$

where $F_n(x_i)$ is the empirical CDF and $F^*(x_i)$ is the CDF of the fitted distribution.

The critical value of the K-S test at significance level 5%, with sample size $n = 1000$ is $\frac{1.36}{\sqrt{1000}} = 0.043$

Since $D < 0.043$, we do <u>not</u> reject the null hypothesis, $H_0: X \sim \text{Exp}(1015.92)$ at the 5% significance level.

(b) (i) The likelihood function is

$$L(\alpha,\theta) = \prod_{i=1}^{n}\frac{x_i^{\alpha-1}e^{-\left(\frac{x_i}{\theta}\right)}}{\theta^{\alpha}\Gamma(\alpha)} = \frac{1}{\theta^{\alpha n}\Gamma(\alpha)^n}\left(\prod_{i=1}^{n}x_i^{\alpha-1}\right)e^{-\sum_{i=1}^{n}\frac{x_i}{\theta}}$$

$$\Rightarrow l(\alpha,\theta) = -\alpha n\ log\ \theta - n\ log\ \Gamma\ (\alpha) + (\alpha - 1)\sum_{i=1}^{n} log\ x_i - \frac{1}{\theta}n\bar{x}$$

$$\Rightarrow \frac{\partial l}{\partial \theta} = -\frac{\alpha n}{\theta} + \frac{n\bar{x}}{\theta^2}$$

Set the derivative equal to 0, and α to $\hat{\alpha}$ for the MLE:

$$\frac{n\bar{x}}{\hat{\theta}^2} - \frac{\hat{\alpha}n}{\hat{\theta}} = 0 \Rightarrow \bar{x} - \hat{\alpha}\hat{\theta} = 0$$

$$\Rightarrow \hat{\theta} = \frac{\bar{x}}{\hat{\alpha}} = \frac{1015.92}{0.92463} = 1098.73$$

(ii) The maximum value of the loglikelihood function is therefore

$$l_{\max} = -\hat{\alpha} n \log \hat{\theta} - n \log \Gamma(\hat{\alpha}) + (\hat{\alpha} - 1) \sum_{i=1}^{n} \log x_i - \frac{1}{\hat{\theta}} n \bar{x}$$

where $\Gamma(\hat{\alpha}) = 1.04955$ from Excel

$$\Rightarrow l_{\max} = -7921.5$$

(iii) The K-S statistic is approximately $D = |-0.014| = 0.014 < 0.043$.

Thus, we do not reject $H_0: X \sim Gamma(0.92463, 1098.73)$ at the 5% significance level.

(c) The AIC and SBC are calculated as follows:

Fitted Distribution	Number of Parameters r	AIC $(l - r)$	SBC $(l - \frac{r}{2} \ln n)$
Exponential	1	-7924.55	-7927.0
Gamma	2	-7923.5	-7928.4

(i) Based on AIC, Gamma is preferred

(ii) Based on SBC, Exponential is preferred

(iii) For the Likelihood Ratio Test of $H_0: X \sim Exp \;\; vs \; H_1: X \sim Gamma$:

The test statistic is

$$T = 2(l_1 - l_0) = 2\big(-7921.5 - (-7923.55)\big) = 4.10$$

Under the null hypothesis, $T \sim \chi^2$ with 1 degree of freedom (difference between the number of parameters.

The p value for this test is

$$p = 1 - F_{\chi^2}(4.1) = 1 - 0.957 \; = 0.043 < 5\%$$

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Therefore, we reject H_0 at the 5% significance level. Gamma is preferred under this test.

(d) Under Exponential,

$P(X > 6000) = e^{-\frac{6000}{1015.92}} = 0.002723$

Under Gamma,

$P(X > 6000) = 1 - F_{Gamma}(6000) = 0.003521$ (using Excel function GAMMA.DIST).

(e) For solvency in reserve calculations, it is important to capture the tail risk of losses (i.e., large claims).

- The gamma distribution passes the K-S goodness of fit test, and is preferred under the AIC and LRT methods.
- We see from (d) that the fitted Gamma distribution assigns a higher probability to claims over 6000 than the exponential and may therefore be a more suitable distribution for capturing tail risk.
- In general, the Gamma distribution with $\hat{\alpha} < 1$ has a relatively heavier tail than the Exponential with the same mean.

Examiners' Comments

Overall, candidates did this question very well.

In (a) (ii), some candidates did not realize that the graph itself has already represented the difference between the empirical distribution function and the fitted exponential distribution function for each data point.

In (b)(ii), some candidates were unable to compute the GAMMA function in Excel.

Candidates did relatively poor in (e). Many candidates stated that the Gamma distribution fitted the data better but failed to explain that the fitted Gamma had a heavier tail based on (d) and worked better for determining margins for solvency.

ASTAM Fall 2023 Q4

(a) (i)

$$L(q) = \prod_{i=1}^{m} q^{I_i}(1-q)^{1-I_i} = q^n(1-q)^{m-n}$$

(ii) $l(q) = n\ln q + (m-n)\ln(1-q)$

Take the derivative of $l(q)$ with respect to q

$$l'(q) = \frac{n}{q} - \frac{m-n}{1-q}$$

Set equal to zero for MLEs

$$\frac{n}{\hat{q}} = \frac{m-n}{1-\hat{q}} \Rightarrow n - \hat{q}n = \hat{q}m - \hat{q}n \Rightarrow \hat{q} = \frac{n}{m}$$

(b) The posterior distribution of q is proportional to the product of the prior density function and the likelihood:

$$\pi_{q|\mathbf{x}}(q) \propto L(q)\,\pi(q) = \left(q^n(1-q)^{m-n}\right)\left(\beta(\beta+1)q(1-q)^{\beta-1}\right)$$

$$\propto q^{n+1}(1-q)^{m-n+\beta-1}$$

This is the kernel of a Beta distribution with parameters $n+2$ and $m-n+\beta$, as required.

(c) The sample mean is n/m, the prior mean is $2/(2+\beta)$, and the Bayes estimate of q under the squared error loss function is the posterior mean of q, that is

$$\frac{n+2}{m+\beta+2} = Z\left(\frac{n}{m}\right) + (1-Z)\frac{2}{\beta+2}$$

$$\Rightarrow \frac{m(n+2)}{m+\beta+2} = Zn + \frac{2m}{\beta+2} - Zm\left(\frac{2}{\beta+2}\right) \quad \text{multiply by } m$$

$$\Rightarrow \frac{m(n+2)(\beta+2) - 2m(m+\beta+2)}{m+\beta+2} = Z(n(\beta+2) - 2m) \quad \text{multiply by } (\beta+2) \text{ and re-arrange}$$

$$\Rightarrow \left(\frac{m}{m+\beta+2}\right)\left(\frac{(n+2)(\beta+2) - 2(m+\beta+2)}{n(\beta+2) - 2m}\right) = Z \quad \text{divide by } n(\beta+2) - 2m \text{ and re-arrange}$$

$$\Rightarrow \left(\frac{m}{m+\beta+2}\right)\left(\frac{n(\beta+2) - 2m}{n(\beta+2) - 2m}\right) = Z$$

$$\Rightarrow \left(\frac{m}{m+\beta+2}\right) = Z$$

(d) (i)

We have:

$$Z_A = \frac{100}{100+20+2} = 0.820$$

$$Z_B = \frac{100}{100+10+2} = 0.893$$

(ii) The Bayes estimates are:

$$\tilde{q}_A = \frac{22}{100+20+2} = 0.180$$

$$or: \quad 0.820(20/100) + 0.180(2/22) = 0.180$$

$$\tilde{q}_B = \frac{12}{100+10+2} = 0.107$$

$$or: \quad 0.893(10/100) + 0.107(2/12) = 0.107$$

(iii) Higher values of β generate lower values for the credibility factor. This means less weight is placed on the estimate from the risk itself, and more weight is placed on the prior estimate. A higher β indicates more confidence in the prior estimate.

A lower value of β means that the prior estimate of q is higher. If we expect a relatively high claim rate, then it makes sense that the credibility factor applied to experience will also be higher.

Examiners' Comments

Overall, this question was poorly done.

Most candidates understand the MLE concept and wrote down the derivative of likelihood function and stated that it should equal to zero, even if the likelihood function wasn't correct written. Most common error was to mix up Binomial probability mass function with the likelihood.

Most candidates simply wrote down the Beta posterior distribution which is given in the question, instead of proving it, even though the question was asking to "show".

For Part (c), using the clue from the question itself, e.g. part (b) the posterior beta mean, and clue from part (c) squared loss function, the credibility factor Z could be solved fairly easily. Unfortunately, the clues from question were ignored by most candidates. A common approach was to calculate from the general form of credibility function (Z=1/(1+v/a)). This approach is do-able but it took too much exam time and almost all of the candidates tried this approach had failed to prove the solution.

For Part (d), most of the candidates did well on this part, if they remembered the formula correctly.

ASTAM Fall 2023 Q5

(a) The aggregate inflation adjusted 2019 AY claims are:

$$5(1.03) + 17(1.03)(1.075) + 38(1.03)(1.075)(1.044) + 104(1.03)(1.075)(1.044)(1.02)$$

$$= 190.53$$

(b) The incremental claims adjusted for inflation to 2023 values are:

	Development Year (DY)			
Accident Year (AY)	0	1	2	3
2019	122.63	43.93	18.82	5.15
2020	122.53	44.29	19.57	
2021	152.80	46.35		
2022	149.35			

The inflation-adjusted cumulative claims are:

	Development Year (DY)			
Accident Year (AY)	0	1	2	3
2019	122.63	166.55	185.38	190.53
2020	122.53	166.82	186.39	
2021	152.80	199.15		
2022	149.35			

The development factors are:

	Development Year (DY)			
	0	1	2	3
$\widehat{f}_j$	1.338	1.115	1.028	-
$\widehat{\lambda}_j$	1.534	1.146	1.028	

Hence, the total incurred claims in 2023 units are:

$$190.53 + 186.39\widehat{f}_2 + 199.15\widehat{f}_2\widehat{f}_1 + 149.35\widehat{f}_2\widehat{f}_1\widehat{f}_0 = 839.4$$

$$or: \ 190.53 + 186.39\widehat{\lambda}_2 + 199.15\widehat{\lambda}_1 + 149.35\widehat{\lambda}_0 = 839.4$$

(c) Assuming policies are issued uniformly over each calendar year:

Year	Earned Premiums	Parallelogram Increase	On-level Factor	Earned Premium at current rates
2019	220	1	1.1024	242.53
2020	225	1.0050	1.0969	246.81
2021	235	1.0370	1.0631	249.83
2022	275	1.0849	1.0162	279.45

The total earned premium for 2019-2022 at current rates is

242.53 + 246.81 + 249.83 + 279.45 = 1018.6

(d) The loss ratio for the period is (839.4 + 35)/1018.6 = 0.8584331

Therefore, the percentage increase in premium is (0.8584331/0.80) – 1 = 0.0730 = 7.30%.

(e) Since there were no premium increases during 2019, the 2019 earned premiums adjusted to current rate levels would stay the same.

Examiners' Comments

This was a question where Execl could be an asset in answering the question. However, as is clearly stated in the instructions, candidates who use Excel MUST provide enough work on the page for the grader to be able to follow their thought process and check their work. There were candidates using Excel who did not provide sufficient information.

There were a number of candidates who did (a) correctly, and then didn't use those values in (b). When developing claims with inflation, the inflation adjustment should be done first, so all values are in a common year, before developing the claims. Many candidates either ignored inflation or calculated development factors using non-inflation-adjusted values and then attempted to adjust for inflation.

Many candidates had trouble with the parallelogram method in part (c).

For part (d), we'd like to emphasize that candidates who used the rounded values in earlier parts of the problem received full credit. Also in (d), the loss ratio needs to cover both claims and fixed expenses and the calculation should involve both earned premium and the developed losses.

For Part (e), a common mistake was to say that issuing policies earlier would cause the EP to increase – a typical reasoning was that there would be more time for inflation to be applied.

ASTAM Fall 2023 Q6

(a)

$$\widehat{f_1} = \frac{\sum_{i=0}^{4-1-1} C_{i,2}}{\sum_{i=0}^{4-1-1} C_{i,1}} = \frac{3120 + 3263 + 3197}{2846 + 3001 + 2797} = 1.1083$$

(b)

$$\widehat{c_{3,4}} = c_{3,1} \cdot \widehat{f_1} \cdot \widehat{f_2} \cdot \widehat{f_3} = 2561 \cdot 1.1083 \cdot 1.0583 \cdot 1.0158 = 3051.2$$

(c)

$$\hat{\sigma}_1^2 = \frac{1}{2}\left\{c_{0,1}\left(f_{0,1} - \widehat{f_1}\right)^2 + c_{1,1}\left(f_{1,1} - \widehat{f_1}\right)^2 + c_{2,1}\left(f_{2,1} - \widehat{f_1}\right)^2\right\}$$

$$= \frac{1}{2}\left\{2846\left(\frac{3120}{2846} - \widehat{f_1}\right)^2 + 3001\left(\frac{3263}{3001} - \widehat{f_1}\right)^2 + 2797\left(\frac{3197}{2797} - \widehat{f_1}\right)^2\right\}$$

$$= 2.5521$$

Therefore, $\widehat{\sigma_1} = \sqrt{2.5521} = 1.5975$

(d)

(i)

The process variance for AY3 is

$$Var\left(C_{3,4}\middle|C_{3,1}\right) \approx \hat{C}_{3,4}^2\left(\frac{\hat{\sigma}_1^2}{\hat{f}_1^2\widehat{C_{3,1}}}+\frac{\hat{\sigma}_2^2}{\hat{f}_2^2\widehat{C_{3,2}}}+\frac{\hat{\sigma}_3^2}{\hat{f}_3^2\widehat{C_{3,3}}}\right)$$

$$= 3051.2^2\left(\frac{2.5521}{1.1083^2\cdot 2561}+\frac{0.4465^2}{1.0583^2\cdot 2838}+\frac{0.1248^2}{1.0158^2\cdot 3004}\right) = 8184.12$$

Therefore, the process standard deviation for AY3 is $\sqrt{8184.12} = 90.47$

(ii)

The estimation error for AY3 is

$$\left(C_{3,4}-E[C_{3,4}|C_{3,1}]\right)^2 \approx \hat{C}_{3,4}^2\left(\frac{\hat{\sigma}_1^2}{\hat{f}_1^2 S_1}+\frac{\hat{\sigma}_2^2}{\hat{f}_2^2 S_2}+\frac{\hat{\sigma}_3^2}{\hat{f}_3^2 S_3}\right)$$

$$= 3051.2^2\left(\frac{2.5521}{1.1083^2\cdot 8644}+\frac{0.4465^2}{1.0583^2\cdot 6383}+\frac{0.1248^2}{1.0158^2\cdot 3284}\right) = 2540.32$$

The square root of the estimation error for AY3 is then $\sqrt{2540.32} = 50.40$

(iii)

The process variance is innate to the underlying claims process and is not related to the estimator. If the parameters were known for certain, the process variance would not change.

The estimation error measures the uncertainty/discrepancy between the true and estimated values of the conditional expected ultimate claims.

Examiners' Comments

Overall, this question was fairly well done. Parts (a) and (b) were particularly done well. For Part (c). the most common mistake was when calculating $f_{i,1}$, many papers used the claim cost from incorrect development years (1 year off). For Parts (d)(i) and (d)(ii), the formulas are provided so the candidates just needed to plug in the correct values. Common errors include miscalculation of $\widehat{f_i}$, $\widehat{C_{3,J}}$, and missed exponent. Part (d)(iii) was not done well. Many candidates omitted this part or wrote down things that were completely irrelevant.