On the regulator-insurer-interaction in a structural model

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Abstract: In this paper, we provide a new insight to the previous work of Briys and de Varenne [1994], Grosen and Jørgensen [2002] and Chen and Suchanecki [2007]. We show that if the insurance company follows a risk management strategy, it can significantly change the risk exposure of the company, and that it should thus be taken into account by the regulators. We first study how the *regulator* establishes regulation intervention levels in order to control for instance the default probability of the insurance company (under the real world probability measure). This part of the analysis is based on a constant volatility and there exists a one-to-one relation between the optimal regulation level and the volatility. Given that the insurance company is informed of the regulatory rules, we study how results can be significantly different when the insurance company follows a risk management strategy with non-constant volatilities. We thus highlight the limits of prior literature and believe that the value of the risk management of the company should be included in the risk exposure estimation and the market value of liabilities as well.

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Journal of Economic Literature Classification: G13, G22, G33

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1 Introduction

The IASB (International Accounting Standards Board) in Europe and the FASB (Financial Accounting Standards Board) in the United States have been working on new accounting standards. It is a “risk-oriented approach”, i.e. assets and liabilities should be evaluated at their market value. This risk-based marketing of products has to include credit risk, market risk and operational risk. There is no market for companies’ liabilities and thus an important need for financial models. An important issue is to address the default risk since a long list of insolvent life insurance companies has been reported since the 1980s in Australia, Europe, Japan and USA[1]. A recent strand of the literature has been developed to model default and market risk, among others Briys and de Varenne [1994, 2001], Grosen and Jørgensen [2000, 2002], Ballotta [2005] and Ballotta, Haberman and Wang [2005].

In fact, the insurance literature on this topic has recently followed step by step the finance works. In the fundamental work of Merton [1974, 1989], risky debt is modeled as riskless debt with a short position in a put option and equity as a call option on the firm’s assets. His closed-form formulae are directly obtained from the well known framework developed by Black-Scholes-Merton [1973]. Following Merton [1974, 1989], Briys and de Varenne [1994, 2001] model default at maturity in an insurance context. Merton’s approach has been then widely extended in finance by Black and Cox [1976] who consider that ruin is possible at any instant. Stochastic interest rates have been then introduced in the previous models by Briys and de Varenne [1997], Longstaff and Schwartz [1995] and Collin-Dufresne and Goldstein [2001] who provide a correction of the latter work. Modeling the default of insurance companies in a Black and Cox framework has been first developed by Grosen and Jørgensen [2002], and then extended by Bernard, Le Courtois and Quittard-Pinon [2005, 2006]. Finally the recent work of Chen and Suchanecki [2007] shows how to apply the realistic procedure Chapter 11 of the US Bankruptcy code in the insurance field[2].

[1]Some exemplary insolvent life insurers are: HIH Insurance in Australian in 2001, Garantie Mutuelle des Fonctionnaires in France in 1993, the world’s oldest life insurer Equitable Life in the United Kingdom in 2000, Mannheimer Leben in Germany in 2003, Nissan Mutual Life in 1997, Chiyoda Mutual Life Insurance Co. and Kyoei Life Insurance Co. in 2000, Tokyo Mutual Life Insurance in 2001, First Executive Life Insurance Co. in 1991 and Conceso Inc. in 2002. It is worth mentioning that First Executive Life Insurance Co. constituted the 12th largest bankruptcy in US during the time period 1980-2005, and Conseco Inc. was the 3rd largest bankruptcy in US for the same period. More details can be found in the recent works of Jørgensen [2004] or Chen and Suchanecki [2007].

[2]The US bankruptcy code distinguishes between Chapter 7 and Chapter 11 bankruptcy procedures. According to Chapter 7 bankruptcy procedure, the default and the liquidation dates coincide. In contrast, Chapter 11 bankruptcy procedure describes a more realistic procedure, in the sense that default and liquidation are distinguishable events. Similar bankruptcy procedures can be found also in France, Germany and Japan etc.

In all the above literature, the emphasis has been laid on the fair valuation of life insurance liabilities under different approaches to modeling default events consistent with the new international accounting standards. Recently, regulatory authorities want to implement a new project in Europe, Solvency II. It has to be compatible with accounting valuation standards and is likely to use measures such as the probability of economic ruin, Value-at-Risk or Tail Value-at-Risk. Insurance companies will be allowed to use internal models to measure their risks. An important issue is thus the robustness of the prior models in the estimation of the risks of the company. In this paper, we are interested in estimating the solvency risks in standard models, for instance with what probability (under the real world probability measure) the insurance company will become bankrupt and which amount they can expect to obtain after taking account of the insurer's default risk. Whilst the insurance company tries to maximize returns for its equity holders, the regulatory authorities are responsible for protecting the policyholders and the stability of the market. Furthermore, in the most of the literature, the regulators act very passively and the role of the regulators is not highly emphasized. However, in reality the collapse of many insurance companies is closely related to the insufficient regulation. For instance, the fall of First Executive Life Insurance Co. provides important lessons in regulation of life insurance companies[3]. Hence, we investigate in particular how the regulator can establish regulatory rules in order to meet some regulatory objectives, like how to control the default probability below a constraint or to keep the expected payoff given liquidation above a certain level. Value-at-risk-based analysis is carried out[4]. Furthermore, Solvency II emphasizes the importance of how to develop new regulatory methods and tools in order to reduce the threat of an insolvency and to better protect the policy holders.

A (hidden) important assumption in the first part of our analysis and in previous literature is that the insurance company follows a risk management strategy with a *constant* volatility. In this framework, we observe a one-to-one relation between the optimal regulation level and the volatility. However, for long-term contracts like life insurance liabilities, it is in fact not realistic to assume that the insurance company follows a risk management strategy with a constant volatility throughout the entire contract period. It indicates that an ex-ante optimal regulation level ceases to be optimal in a realistic model setup where the insurance company follows a risk management strategy with another volatility. A"fixed volatility rule" becomes useless whenever the insurance company follows a strategy with a non-constant volatility. We highlight the limitations of this "fixed volatility" assumption common to all the above references. In the second part of our analysis, we release this assumption and investigate how the results from the first part are influenced when the

[3]C.f. Schulte [1991] provides an insider's view on the fall of First Executive Life Insurance Co.

[4]Basak and Shapiro [2001] investigate the influences of VaR-based regulation on the institutional investors' risk management strategies.

insurance company is informed of regulators' rules and reacts to the rule according to a certain discrete risk management strategy with non-constant volatilities. By introducing this simple strategy with a non-constant volatility, noticeable effects are observed. A distinguishable difference of our analysis in comparison with the existing literature is that the role of the regulator is strengthened and the interaction between the regulator who determines the regulation rule and the insurance company's risk management is highlighted.

The remainder of the paper is structured as follows. Section 2 is devoted to the formulation of the problem in the context of solvency requirements. Section 3 demonstrates two different ways to model default risk and calculate default probability in each case in a static framework. Due to the constraints of the static framework, in section 4 we extend it to a dynamic framework in which the company is allowed to have a strategy with a non-constant volatility. We highlight the problem of the robustness of the previous results from a regulatory perspective. The last section concludes.

2 Problem

We study the regulation of a company that would sell only one type of contract. Given no early default of the insurance company, these products ensure their holders a minimum guaranteed amount at maturity and a participation in the surpluses of the company, if any. Whereas if default arises at a premature time, a rebate payment which is a function of the regulation level is offered to the policyholders.

We consider two issues in the following of this paper.

- The first part of our analysis (section 3) persists in the regulators' viewpoint. The regulators want to determine the optimal level of intervention (i.e. optimal barrier level) in order to protect the policyholders. They will choose the default level in order to keep some fixed limit, e.g. to have a probability of default less than 0.5%. This part of the analysis is said to be a "static" framework.
- A main result from the static analysis is that the optimal regulation (or barrier) level is linked to a "fixed-volatility" rule. However, in a more realistic framework, the "fixed-volatility" rule is not satisfied, when the insurance company adjusts its risk management strategy (in the sense of adjusting the volatility of the strategy) during the entire contract period. The ex-ante optimal barrier ceases to be optimal. Therefore, we show the limitations of the existing literature where a fixed volatility is applied. In the second part of our analysis (section 4), we move to a "dynamic" framework, where the insurance company trades in a risk management strategy with a non-constant volatility. Through some comparative statistics between the static and

dynamic framework, we observe significant differences between these two approaches.

2.1 Adopted framework

A "structural model" for the default means that the default is directly triggered by the observation of the firm's assets (for instance Merton [1974] and Black and Cox [1976]). These models were first applied to insurance by Briys and de Varenne [1994, 2001] (no early default possible) and Grosen and Jørgensen [2002] (premature default possible). In these models, a representative liability holder pays an upfront premium which corresponds to an α-fraction of the entire company's initial assets. Accordingly, the equity holder's contribution corresponds to $(1-\alpha)$-fraction of the initial assets' amount. The policyholders all invest in the same contract maturing at time T, guaranteeing a minimum interest rate g and a participation rate δ.

We make the standard assumptions of the Black and Scholes framework. First there exists a risk-free asset with a continuous constant interest rate r. Trading takes place continuously. There are no tax, no transaction costs and no agency costs. Moreover there are no dividends. More importantly, the analysis of sections 2 and 3 is based on the assumption of a fixed volatility level and for convenience we would use "static" framework to describe this assumption. Throughout this paper, we use the following notations:

r	:	constant risk-free interest rate
σ	:	constant assets' volatility
T	:	the contract's maturity date
$L_T = L_0 e^{gT}$	:	the guaranteed payment to the policy holder at maturity, where g could be interpreted as the minimum guaranteed interest rate
L_t	:	the minimum guarantee of the insured's investment at time $t \in [0,T]$
A_t	:	the value of the firm's assets at time $t \in [0,T]$
B_t	:	ηL_t, the barrier level, where η is the regulation parameter.

As compensations to their initial investments in the company, equity holders and policyholders both acquire a claim on the firm's assets for a payoff at maturity if no premature default occurs. The total payoff to the holder of such an insurance contract at maturity, $\psi_L(A_T)$, is given by:

$$\psi_L(A_T) = L_T + \delta[\alpha A_T - L_T]^+ - [L_T - A_T]^+. \tag{1}$$

This payment consists of three parts: the guaranteed amount at maturity (a guaranteed fixed payment which is the initial premium payment compounded by the interest rate guarantee), a bonus (call option) paying to the policy holder a fraction δ of the positive difference of the actual performance of his share in the insurance company's assets, and a short put option resulting from the fact that the equity holder has limited liabilities.

It is noticed that the incentives for customers to buy this kind of contracts are not very high for two reasons: first, the guaranteed interest rate is usually smaller than the market interest rate; and second, probably the customers cannot obtain the guaranteed amount which is against the nature of an insurance contract. Therefore, it seems more interesting to analyze the risk management of these contracts than to price them. We thus analyze different risk measures under the real world measure instead of under the equivalent martingale measure.

2.2 Default Modeling

For the default modeling, we examine two different scenarios: Grosen and Jorgensen [2002] and Chen and Suchanecki [2007]. In both works, the default barrier is monitored continuously. The continuous surveillance makes sense because a company has to be solvent at any time. It would not be interesting to consider only Merton's case where solvency is required at maturity only. One reason could also be that most of the time such policies include some surrender options, meaning that the policyholder can claim at any time for his investment. The company should then be able to give back the promised amount. In the case of Grosen and Jørgensen, as soon as the level of the assets is not sufficient to fulfill its commitments, the regulator liquidates the company immediately. Whereas in Chen and Suchanecki [2007], the company is not immediately liquidated when its assets hit the fixed level, but it is given a period of time to recover before its liquidation by the regulators. If the US Bankruptcy Code is taken as an example, Grosen and Jørgensen [2002] corresponds to a Chapter 7 bankruptcy procedure while in Chen and Suchanecki [2007], a Chapter 11 bankruptcy procedure is analyzed where default and liquidation are considered to be distinguishable events.

Grosen and Jørgensen [2002] model their regulatory intervention rule in the form of a boundary, i.e., an *exponential* barrier $B_t = \eta L_0 e^{gt}$ is imposed on the underlying assets value process, where η is a regulation parameter. When the asset price reaches this boundary, namely, $A_\tau = B_\tau$ with $\tau = \inf\{t \in [0,T] | A_\tau = B_\tau\}$ denoting the first passage time, the company defaults and is liquidated immediately, i.e., default and liquidation coincide. In addition, in Grosen and Jørgensen [2002], the default time τ coincides with the liquidation date and upon liquidation a rebate payment,

$$\min\{L_0 e^{g\tau}, B_\tau\} = \min\{1, \eta\} L_0 e^{g\tau}, \tag{2}$$

is offered to the liability holder.

Chen and Suchanecki [2007] model a Chapter 11 bankruptcy procedure by using both standard and cumulative Parisian down-and-out frameworks. The standard Parisian barrier feature corresponds to a procedure where the liquidation of the firm is declared when the financial distress has lasted successively at least a period of length d. The cumulative Parisian barrier feature corresponds to a procedure where the liquidation is declared when the financial distress has lasted in total at least a period of length d during the life of the contract. The economic idea behind these two extremes is the importance of the past of the company's assets. Indeed in the standard Parisian case regulators completely forget the past. On the contrary the cumulative procedure corresponds to the extreme case when the regulators never forget the past. In the standard case, an early liquidation occurs when the following technical condition is satisfied:

$$T_B^- = \inf\left\{t > 0 \middle| \left(t - g_{B,t}^A\right) \mathbf{1}_{\{A_t < B_t\}} > d\right\} \leq T$$

with

$$g_{B,t}^A = \sup\{s \leq t | A_s = B_s\},$$

where $g_{B,t}^A$ denotes the last time before t at which the value of the assets A hits the barrier B. T_B^- gives the first time at which an excursion below B lasts more than d units of time. In fact, T_B^- is the early liquidation date of the company if $T_B^- < T$.

In case of the cumulative Parisian framework, the options are lost by their owners when the underlying asset has stayed below the barrier for at least a predetermined d units of time during the entire duration of the contract. Therefore, an early liquidation when the following condition holds:

$$\Gamma_T^{-,B} = \int_0^T \mathbf{1}_{\{A_t \leq B_t\}} dt \geq d,$$

where $\Gamma_T^{-,B}$ denotes the occupation time of the process describing the value of the assets $\{A_t\}_{t\in[0,T]}$ below the barrier B during $[0, T]$. Again, we denote τ as the premature liquidation date and it implies:

$$\Gamma_\tau^{-,b} := \int_0^\tau \mathbf{1}_{\{\tau \leq T\}} \mathbf{1}_{\{A_t \leq B_t\}} dt = d.$$

3 Optimal barrier under continuous surveillance

In this section, instead of emphasizing the fair valuation of life insurance liabilities under different default triggers as most of the existing literature does, we study risk measures and therefore adopt the regulators' viewpoint. In addition, in the formulation of section 2.1, the regulator does not play a very important role. As a starting point, we set ourselves in Grosen and Jørgensen's framework and assume that the regulator plays a multiple role, i.e., he strives not only for a low default probability of the company but also aims at providing the policyholder a certain amount in case of default. Mathematically, the two goals of the regulator are given by:

$$\min \{\eta > 0 \ / \ P(\tau \leq T) \ \leq \ \varepsilon \} \tag{3}$$

$$\max \left\{\eta > 0 \ / \ E\left[\min\{\eta, 1\} L_\tau \, e^{r(T-\tau)} | \tau \leq T\right] \geq \gamma L_T \right\}, \tag{4}$$

The goal of the regulator is to find a regulation parameter which first gives an acceptable level of default and second protects the policyholders by maximizing their expected cash flows given default. Some comments should be made concerning the second aim: (a) Given default, the policyholder obtains the rebate term, which corresponds to the term $\min\{\eta, 1\}L_\tau$; (b) In order to make it comparable with the final payment, the rebate payment is accumulated to the maturity date with the risk-free interest rate r; (c) $\gamma \in [0, 1]$ implies that the regulator sets the regulation rule to provide in expectation γ percents of the final guaranteed payment to the policyholder.

3.1 Aim 1: Minimizing the default probability

In order to compute the default probability and the expected value, the firm's assets value is assumed to follow a geometric Brownian motion under the real world measure P

$$dA_t = A_t \, (\mu \, dt + \sigma \, dW_t),$$

where μ and $\sigma > 0$ are respectively the instantaneous rate of return and the volatility of the assets. W_t is assumed to be a standard Brownian motion under the real world measure P.

We begin with the first goal given in Equation (3), i.e. to compute the probability that an early default occurs: $P(\tau \leq T)$. According to the derivation of Appendix A,

$$P\left(\tau \leqslant T\right) \ = N\left(\frac{\ln(\frac{\eta L_0}{A_0}) - \hat{\mu} T}{\sigma \sqrt{T}}\right) + \left(\frac{A_0}{\eta L_0}\right)^{\frac{-2\hat{\mu}}{\sigma^2}} N\left(\frac{\ln(\frac{\eta L_0}{A_0}) + \hat{\mu} T}{\sigma \sqrt{T}}\right)$$

with $\hat{\mu} = \mu - g - \frac{\sigma^2}{2}$.

Table 1 demonstrates several cumulative default probabilities for a time horizon of 20 years. According to e.g. Moody's credit rating, a small volatility of 10% leads to a very small default probability and this leads to an Aaa rating of the company, a volatility value

σ	10%	15%	20%
$P(\tau \leqslant T)$	0.00257218	0.07269	0.239842

Table 1: Cumulative default probabilities with parameters: $A_0 = 100$; $L_0 = 80$; $T = 20$; $\eta = 0.5$; $\mu = 0.04$; $r = 0.03$; $g = 0.01$.

of 15% results in Baa[5].

Thanks to its definition, one can easily notice that the probability of default is a strictly nondecreasing function with respect to the variable η. The higher the η, the higher the default probability. Therefore η^{ε} is the unique solution satisfying the equation:

$$N\left(\frac{\ln(\frac{\eta^{\varepsilon}L_0}{A_0}) - \hat{\mu}T}{\sigma\sqrt{T}}\right) + \left(\frac{A_0}{\eta^{\varepsilon}L_0}\right)^{\frac{-2\hat{\mu}}{\sigma^2}} N\left(\frac{\ln(\frac{\eta^{\varepsilon}L_0}{A_0}) + \hat{\mu}T}{\sigma\sqrt{T}}\right) = \varepsilon.$$

When the regulator sets a regulation parameter smaller than this critical value, i.e.,

$$\eta \ \leq \ \eta^{\varepsilon}, \tag{5}$$

a default probability smaller than ε can be achieved. Figure 1 demonstrates how the optimal regulation parameter depends on the constrained default probability ε for different volatility values. The higher the ε-value, the higher the resulting optimal regulation parameter. Furthermore, the higher the volatility, for a given ε-value, the lower the optimal regulation parameter. In addition, it is observed that a quite low regulation parameter η which results in a quite low barrier level should be chosen in order to keep the insurance company to at (or below) a reasonable default probability.

3.2 Aim 2: Maximizing the expected payout of the contract given liquidation

We proceed with the second goal. The regulator wants to maximize the expected conditional cash flows of the insured with respect to η. In Grosen and Jørgensen [2002], the rebate payment is paid out immediately at the premature liquidation time. For compatibility reasons, it is assumed now that the rebate payment will be accumulated with a risk-free market interest rate and paid out at the maturity. According to the derivation in

[5] According to Moody's rating, for a 20-year horizon, the ratings and the respective default probabilities are given as follows: Aaa, 1.55%; Aa, 2.70%, A, 5.24% and Baa, 12.59%.

Optimal Regulation Parameter in Case of Grosen and Jørgensen

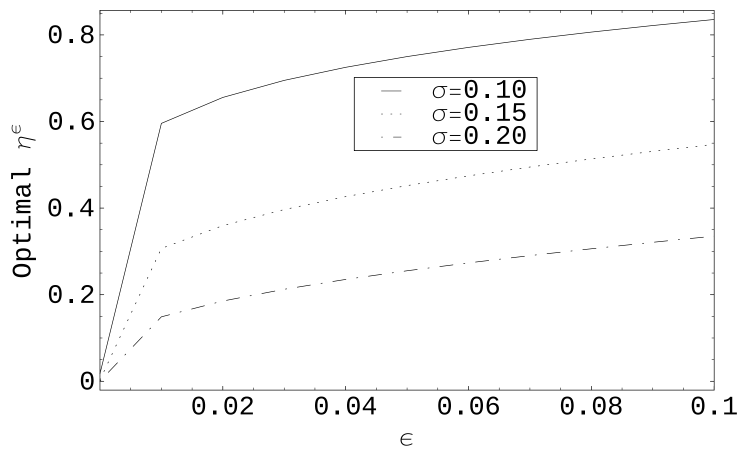

Figure 1: Trade-off between η^{ε} and ε when parameters are set to: $A_0 = 100$; $L_0 = 80$; $T = 20$; $\mu = 0.04$; $r = 0.03$; $g = 0.01$.

ε	η^{ε}; $\sigma = 0.10$	η^{ε}; $\sigma = 0.15$	η^{ε}; $\sigma = 0.20$
0	0	0	0
0.01	0.595660	0.306855	0.148879
0.02	0.655581	0.359548	0.185358
0.03	0.694975	0.396648	0.212528
0.04	0.725144	0.426470	0.235245
0.05	0.749929	0.451935	0.255261
0.06	0.77114	0.474452	0.273434
0.07	0.789786	0.494819	0.290258
0.08	0.806489	0.513537	0.306044
0.09	0.821664	0.530945	0.321006
0.10	0.835603	0.547280	0.335295

Table 2: Optimal regulation parameters η^{ε} for given default probability constraint ε for diverse σ-values with parameters: $A_0 = 100$; $L_0 = 80$; $T = 20$; $\mu = 0.04$; $r = 0.03$; $g = 0.01$.

Appendix B, the expected payoff given liquidation is described by

$$\begin{aligned}
&\frac{E\left[(\eta \wedge 1)\, L_0 e^{g\tau} e^{r(T-\tau)} \mathbf{1}_{\{\tau<T\}}\right]}{P\{\tau \leq T\}} \\
&= \quad (\eta \wedge 1)\, L_0 e^{rT} \Bigg\{ \left(\frac{\eta L_0}{A_0}\right)^{\frac{\hat{\mu}}{\sigma^2} - \frac{\sqrt{(\hat{\mu})^2 + 2(r-g)\sigma^2}}{\sigma^2}} N\left(\frac{\ln(\frac{\eta L_0}{A_0}) - \sqrt{(\hat{\mu})^2 + 2(r-g)\sigma^2} T}{\sigma\sqrt{T}}\right) \\
&\quad + \left(\frac{\eta L_0}{A_0}\right)^{\frac{\hat{\mu} + \sqrt{\hat{\mu}^2 + 2(r-g)\sigma^2}}{\sigma^2}} N\left(\frac{\ln(\frac{\eta L_0}{A_0}) + \sqrt{(\hat{\mu})^2 + 2(r-g)\sigma^2} T}{\sigma\sqrt{T}}\right) \Bigg\} \Big/ P(\tau \leq T)
\end{aligned}$$

with the denominator corresponding to the result in the first aim.

The goal of the regulator here is to provide the policyholder in expectation with a certain γ-fraction of the guaranteed amount given liquidation. Hence, we use η^γ to denote the critical η-value which makes the above expected value equal to $\gamma\, L_T = \gamma\, L_0 e^{gT}$, then

$$\eta \;\geq\; \eta^\gamma \tag{6}$$

provides in expectation that the payoff given liquidation is not smaller than a γ percentage of the guaranteed payment. This is due to the fact that the expected rebate payment goes up with the regulation parameter η. This positive relation between γ and η^γ is observed in Figure 2.

Obviously, there is a trade-off between these two goals. A small default probability requires a lower value of η, but at the same time leads to a lower expected rebate payment which is probably even lower than the γ-fraction of the final guaranteed payment. Therefore, a regulator has to set a regulation level in the area of

$$\eta^\gamma \;\leq\; \eta \;\leq\; \eta^\varepsilon \tag{7}$$

in order to achieve a plausible default probability and to ensure a reasonable expected payoff to the policyholder given default. However, comparing the the optimal values of η^ϵ and η^γ given in Tables 2 and 3, seldom intersection areas of η as in Equation (7) can be found, i.e., usually the regulator cannot aim at both of the goals. This is why in practice most of the regulators stick to the first objective, i.e. to control the default probability under a certain constraint. In the remaining section of this paper, we follow this convention and focus on the first goal. In the following, we examine whether the result changes a lot when a more realistic bankruptcy procedure, i.e. Chapter 11 comes into consideration.

3.3 Under Chapter 11

The purpose of this subsection is to examine whether the realistic bankruptcy procedure Chapter 11 brings some new aspects to our analysis, therefore, we jump to the numerical results immediately. Those who are interested in the derivation of the default probability can have a look at Appendix C.

Tables 4 and 5 demonstrate several optimal values of the regulation parameters for both the standard and cumulative Parisian option case. Above all, it is observed that the resulting optimal η^ε-values are higher than the results in section 3.1. This is quite obvious, because default does not result in liquidation immediately by taking account of a Chapter

γ	η^γ; $\sigma = 0.10$	η^γ; $\sigma = 0.15$	η^γ; $\sigma = 0.20$
0.70	0.607954	0.584077	0.566748
0.75	0.643793	0.619084	0.60125
0.80	0.678647	0.653348	0.635153
0.85	0.712546	0.686897	0.668484
0.90	0.745526	0.719758	0.701264
0.95	0.777624	0.751958	0.733516
1.00	0.808877	0.783522	0.765261

Table 3: Optimal regulation parameters η^γ for diverse γ and σ-values with parameters: $A_0 = 100$; $L_0 = 80$; $T = 20$; $\mu = 0.04$; $r = 0.03$; $g = 0.01$.

Expected payment given liquidation

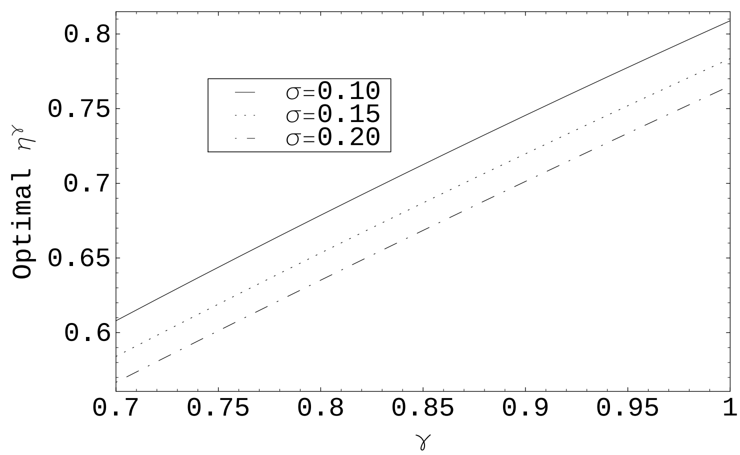

Figure 2: Expected conditional payment given liquidation with parameters $A_0 = 100$; $L_0 = 80; T = 20; r = 0.03;\ \mu = 0.04; g = 0.01$.

11 bankruptcy procedure. Furthermore, the resulting optimal regulation parameters in the standard Parisian framework are higher than those in the cumulative case. This is due to the fact that the knock-out condition in the standard case occurs with a lower probability than in the cumulative case if the same parameters are assumed. The knock-out condition for standard Parisian barrier options is that the underlying asset stays *consecutively* below the barrier for a time longer than d before the maturity date, while the knock-out condition for cumulative Parisian barrier options is that the underlying asset value spends until the maturity *in total* d units of time below the barrier.

However, the positive relation between the volatility and η^ε is still observed. Conse-

ε	η^ε; $\sigma = 0.10$	η^ε; $\sigma = 0.15$	η^ε; $\sigma = 0.20$
0.01	0.6536	0.35281	0.17954
0.02	0.7178	0.413186	0.223563
0.03	0.7603	0.45543	0.25576
0.04	0.7922	0.48964	0.28365
0.05	0.82015	0.51821	0.307534
0.06	0.8443	0.54312	0.32928
0.07	0.8659	0.56654	0.349880
0.08	0.8827	0.58754	0.36734
0.09	0.8991	0.6076	0.3855321
0.10	0.9156	0.62735	0.401856

Table 4: Optimal regulation parameters η^ε in case of standard Parisian option for given default probability constraint ε for diverse σ-values with parameters: $A_0 = 100$; $L_0 = 80$; $T = 20$; $\mu = 0.04$; $r = 0.03$; $g = 0.01$; $d = 0.5$.

quently, introducing the more realistic bankruptcy modeling does not lead to much more different results.

ε	η^ε; $\sigma = 0.10$	η^ε; $\sigma = 0.15$	η^ε; $\sigma = 0.20$
0.01	0.6332	0.33756	0.16965
0.02	0.69658	0.39485	0.210678
0.03	0.73819	0.43545	0.24126
0.04	0.77004	0.46778	0.266954
0.05	0.796205	0.485654	0.28935
0.06	0.81878	0.520094	0.30984
0.07	0.838412	0.54217	0.32865
0.08	0.855952	0.56254	0.34637
0.09	0.87200	0.581354	0.363189
0.10	0.88692	0.59997	0.3791764

Table 5: Optimal regulation parameters η^ε in case of cumulative Parisian option for given default probability constraint ε for diverse σ-values with parameters: $A_0 = 100$; $L_0 = 80$; $T = 20$; $\mu = 0.04$; $r = 0.03$; $g = 0.01$; $d = 0.5$.

3.4 Critiques of the static approach

Whether we are in Grosen or Jørgensen's [2002] or Chen and Suchanecki's [2007] framework, the optimal regulation rule depends on the level of the volatility of the assets, i.e., there exists an interplay between the insurer and the regulator. If the insurer switches his risk management portfolio to another one with a different volatility, the originally optimal regulation parameter loses its optimality. This implies that the insurer can readjust its portfolio to avoid the intervention of the regulator, given that he is fully informed about the regulatory objectives.

Now let us assume that the regulation parameter η is given $\eta = 0.8$. The company is assumed to be perfectly aware of this coefficient, then it has an optimal volatility level or debt ratio level in order to avoid an intervention of the regulators with probability 99%. All parameters being fixed except the assets' volatility.

	Grosen and Jørgensen	Standard Parisian	Cumulative Parisian
Volatility Level	0.0752	0.0817	0.07945

Table 6: Volatility Level $A_0 = 100$; $L_0 = 80$; $T = 20$; $\mu = 0.04$; $r = 0.03$; $g = 0.01$; $d = 0.5$.

Table 6 shows the *maximal* volatility level the insurer can choose in order to avoid the regulator's intervention. In addition, the company may choose a riskier portfolio in the case of a Parisian surveillance for a given level of bankruptcy risk. The probability to have an intervention is indeed lower in the Parisian setting than in a Grosen and Jørgensen setting.

	Grosen and Jørgensen	Standard Parisian	Cumulative Parisian
Debt ratio α ($\sigma = 0.10$)	0.59566	0.65262	0.63329
Debt ratio α ($\sigma = 0.15$)	0.306855	0.35497	0.337397

Table 7: $A_0 = 100$; $T = 20$; $\mu = 0.04$; $r = 0.03$; $g = 0.01$; $d = 0.5$; $L_0 = \alpha A_0$.

Table 7 gives results when all parameters are fixed except the debt ratio α, meaning that the company will ask for more capital to decrease its risk. The listed α value in this table is the *maximal* debt ratio the insurer is allowed to own such that no regulatory intervention is going to take place. For a given bankruptcy risk level, Table 7 illustrates the fact that the company needs more capital in a Grosen and Jørgensen setting than in a Parisian setting. For example when the volatility is set at 15%, the shareholders part should represent 69.3% in a Grosen and Jørgensen setting instead of 64.5% in a Parisian setting to avoid a regulatory intervention with probability 99%.

We can see in Tables 6 and 7 that in the three cases, the results are nearly the same. This shows that the model is robust with respect to the choice of the default model. In the next section, we will see that it is not robust to the assumption made on the dynamic of the assets (which is a stationary model).

4 Moving to a dynamic approach

We keep assuming a constant interest rate and the firm's asset follows a geometric Brownian motion. In the previous section, we assumed that the volatility is constant and this means implicitly that the company follows a risk management strategy with a fixed volatility. This might be reasonable for short-term contracts, but the life insurance contracts considered here are often long-term contracts with a maturity T equal to e.g. 20 years. During such a long-term contract period, it is very likely that the insurance company readjusts its risk management to another volatility. In addition, the analysis in section 3.4 implies that the insurance company might do this on purpose in order to avoid the regulator's intervention. Therefore, in this section, we release the assumption of a constant volatility, i.e. the insurer does not stay passive until the maturity any more. He does react to the regulator's regulation rule and adjusts his risk management strategy (especially allowed to change its volatility). We know from the above study that the optimal regulation rule cannot remain optimal when the insurer changes the volatility, and the resulting default probability might not be promising to the regulator any more. Hence, the regulator is forced to adjust the regulation level, the insurer will then react to that again and so forth. In the following, we call this approach where the insurer follows a risk-switching strategy "a dynamic approach". In this approach, both the insurer and the regulator become more active. We mainly study the impacts of the dynamic approach and accordingly highlight the importance of the interaction between the regulator and the insurer.

In the following, we introduce a very simple strategy where the insurance company can choose strategies with different volatilities and examine what an impact this simple dynamic risk management strategy has on the expected return and on the default probability. We bring the expected return into play to gain more insights from the dynamic approach and it is defined specifically later. That means, in addition to the default probability, the expected rate of return is used as a criterion to compare the static and dynamic approach. As proposed by Dangl and Lehar [2004] for a bank, we assume there are two different portfolios with two levels of asset risk. At the end of each year before the maturity of the contract, four different events might occur:

- The regulators look at the value of the assets of the company and declare bankruptcy because it is too low.
- The company is solvent but too risky: regulators switch the level of asset risk to a lower level to satisfy the regulatory constraints.
- Given the regulatory requirements, it is optimal for the managers to stick to the current risk level.
- Given the regulatory constraints, it is optimal to switch the level of asset risk. In that case that means either company performs well and can take more risk or the company wants to avoid a future regulatory intervention.

4.1 Risk and returns in the static framework

We assume that the parameters are set to $\mu_L = 4\%$, $r = 3\%$ and $\sigma_L = 10\%$ in the lower asset risk case. In the higher asset risk case, we assume $\sigma_H = 20\%$ and $\mu_H = 5\%$[6]. At time 0, the volatility is set at $\sigma = \sigma_0$ and the ratio $\alpha = \alpha_0 = 0.4$, $A_0 = 100$, $L_0 = 40$, $T = 20$, $r = 0.03$, $g = 0.01$, $\delta = 90\%\,\hat{\delta}$. Some comments shall be made concerning the participation rate δ with which the liability holder is allowed to participate in the surpluses of the insurance company. First, δ is not important in the previous analysis and becomes relevant in the analysis of the expected return. Second, $\hat{\delta}$ denotes the participation rate which makes the considered contract fair.[7] Following the ideas of Boyle and Tian [2006], we use a δ-value lower than the fair value to take account of the safety loading in the pricing of equity-linked life insurance. As an example, we choose $\delta = 90\%\,\hat{\delta}$.

We have closed-form expression of the probability of a regulators' closure decision before the maturity T in case of continuous monitoring if the initial assets risk is set at σ_0 (c.f. section 3). We assume the level of bankruptcy at time t is given by ηL_t where $\eta = 0.4$. We define the expected return of the policyholders as:

$$\frac{\text{Expected Payoff at time } T \quad - \quad L_0}{L_0},$$

where the expected payoff of policyholders is given by

$$E[\left(\delta[\alpha A_T - L_T]^+ + L_T - [L_T - A_T]^+\right) \mathbb{1}_{\{\tau \geq T\}}] + E[e^{r(T-\tau)} \min\{A_\tau, L_\tau\} \mathbb{1}_{\{\tau < T\}}].$$

[6]The choice of μ_L, σ_L and that of μ_H, σ_H lead to the same ratio.

[7]The fair participation rate $\hat{\delta}$ results from the fair valuation principle, i.e.

$$E^*[e^{-rT}\left(\hat{\delta}[\alpha A_T - L_T]^+ + L_T - [L_T - A_T]^+\right) \mathbb{1}_{\{\tau \geq T\}}] = L_0,$$

where E^* represents the expectation taken under the equivalent martingale measure.

The rebate payment is again accumulated with the risk free interest rate to maturity date T. In the case of prior bankruptcy, we assume shareholders receive nothing and the assets are used to pay the bankruptcy costs and to reimburse the policyholders. The expected return of the shareholders is given by:

$$\frac{\text{Expected Payoff at time } T \quad - \quad (1-\alpha)A_0}{(1-\alpha)A_0}.$$

	$\sigma_0 = \sigma_L = 10\%$, $\mu_L = 4\%$	$\sigma_0 = \sigma_H = 20\%$, $\mu_H = 5\%$
Default Probability	$3.8 \cdot 10^{-4}\%$	10.8%
Policyholders Expected Return	1.06	1.25
Shareholders Expected Return	1.34	2.00

Table 8: Default probability and expected return in a static setting.

We fix a maximum level of risk (through a given default probability) and use the volatility parameter to adjust the portfolio in order to maximize the expected payments to shareholders and policyholders keeping a default probability prior to maturity below a maximum level.

4.2 Risk and returns in the volatility switching model

Given a maximum probability of bankruptcy before maturity T, denoted by $p_0 = P_0(T)$ (for example 4%) then the company wants to maximize the shareholders' value keeping the probability of an early closure below p_0. The insurance company switches the portfolio at the end of each year as long as no early default occurs. At the end of each year $t = t_i$ ($i = 1, \cdots, T$), managers face three different situations:

-*Case 1:* $A_t < B_t$ Bankruptcy is declared, shareholders receive nothing and policyholders receive A_t.

-*Case 2:* $A_t \geq B_t$ and $\sigma = \sigma_H$. We then compute at time t, the probability of bankruptcy before T when there is no switching until T (we use closed-form formulae provided in section 3). If this default probability is above p_0, then regulators reduce the level of the volatility, otherwise they do not intervene.

-*Case 3:* $A_t \geq B_t$ and $\sigma = \sigma_L$. The managers decide to switch to a higher risk level in order to increase their expected payment. Their decision should keep on satisfying that bankruptcy probability before maturity is below p_0.

We proceed by Monte Carlo methods assuming the initial volatility is either $\sigma_0 = 10\%$ or $\sigma_0 = 20\%$.

	$\sigma_0 = \sigma_L = 10\%$, $\mu_L = 4\%$	$\sigma_0 = \sigma_H = 20\%$, $\mu_H = 5\%$
Default Probability	0.62%	0.64%
Policyholders Expected Payment	1.40 (+32%)	1.15 (-8%)
Shareholders Expected Payment	1.69 (+26%)	1.86 (-7%)

Table 9: Default probability and expected payments in case of dynamic approach. In parenthesis, we give the percentage of increase or decrease compared to the situation with static case.

Through results displayed in Table 8, we identify two different situations whether the company bears initially a high asset risk or a low asset risk. First if it has a low investment risk (initial volatility is set to 10%) then in the static framework, returns are rather small (1.06 for policyholders and 1.34 for shareholders). If it follows the above simple strategy, then the default probability is higher but remains acceptable and expected returns are more interesting and realistic. Secondly, if the company is initially very risky, then the simple switching strategy strongly reduces the risk. Indeed in the static approach the default probability is 10.8% which is not realistic. This simple strategy decreases its default probability to 0.64%. The expected returns are a bit lower but not significantly lower. Therefore, our dynamic approach can then have two interesting effects, i.e. either it increases the expected payments without changing significantly the probability of default or it decreases significantly the default probability keeping rather interesting expected returns.

We are aware that this strategy is an over-simplified example but it already shows the possible impact of a dynamic approach. Indeed, a simple strategy which allows for a non-constant volatility, even through a very simple example, changes the results strongly.

5 Conclusion

Instead of making fair valuation analysis of equity-linked life insurance contracts under consideration of default risk, we mainly look at risk measures under the market measure. More precisely, we explain how the optimal level of intervention (i.e. optimal barrier level) can be determined in order to reach some regulation rules and finally to protect the policyholders. We consider a "fixed–volatility" rule in determining the optimal regulation level and observe that there exists an interaction between the regulator and the insurer. The second part of our analysis focuses then on how the results are influenced when the "fixed-volatility" assumption is lifted, i.e. when the insurance company has a strategy with a non-constant volatility and when the interaction between the regulator and the insurer is taken into consideration.

The fact that the insurance company has a strategy, even a very simple strategy can significantly change the results. Regulators and insurers who implement internal models to value liabilities and to measure risks should be aware that a "static" model ignores the strategy of the managers and will significantly underprice the contracts and overestimate the risks. In this study, we look at the risks estimators (under the historic measure) but a straightforward numerical example can show that the market value of the contracts (and the fair values of the parameters) highly depend on the insurer's strategy. We thus show some limitations of most of the existing literature in this field which assume a given dynamic with a fixed volatility for the whole life of the contract, even if it is a long-term contract.

As starting point for further research, we note that surveillance has a cost. The above study describes continuous monitoring of the assets' process. Following Merton [1978] who extended his previous work by introducing random and costly audits, or the recent works of Battachaya *et al.* [2002] or Dangl and Lehar [2004] one may extend our framework to the case of random audits. The default barrier would not be continuous any more and the company has to be solvent at any time an audit takes place. Furthermore, in this analysis, "non-market risks" are completely ignored in the geometric Brownian motion setting. Therefore, taking into account the non-hedgeable non-market risk like insurance risk is an important extension to the problem.

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Appendix

A Derivation of the default probability

It is observed that

$$A_u \geq B_u \Leftrightarrow \left(\mu - \frac{1}{2}\sigma^2 - g\right) u + \sigma W_u + \ln\left(\frac{A_0}{B_0}\right) > 0$$

Hence, under the market measure P, passage of $A(.)$ through $B()$ is equivalent to the passage of the Brownian motion $Z_u = \left(\mu - \frac{1}{2}\sigma^2 - g\right) u + \sigma W_u + \ln\left(\frac{A_0}{B_0}\right)$ through zero. Now we assume that

$$\hat{\mu} = \mu - g - \frac{1}{2}\sigma^2 \quad ; \quad Z_0 = \ln\left(\frac{A_0}{B_0}\right)$$

consequently

$$\begin{aligned} A_T &= A_0 e^{(\mu - \frac{1}{2}\sigma^2)T + \sigma W_T} = B_0 e^{Z_T + gT} \\ A_T &> B_T \Leftrightarrow Z_T > 0. \end{aligned}$$

Furthermore, it is known that the density of the first passage time is $g(\tau, Z_0, 0) = \frac{Z_0}{\sigma \tau^{3/2}} n\left(\frac{Z_0 + \hat{\mu}\tau}{\sigma\sqrt{\tau}}\right)$, with $n(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$. And the distribution function of τ is

$$G(\tau, Z_t, t) = N\left(\frac{-Z_t - \hat{\mu}(\tau - t)}{\sigma\sqrt{\tau - t}}\right) + e^{-\frac{2Z_t\hat{\mu}}{\sigma^2}} N\left(\frac{-Z_t + \hat{\mu}(\tau - t)}{\sigma\sqrt{\tau - t}}\right)$$

Hence, the default probability we look for is given by:

$$\begin{aligned} P^\varepsilon &= N\left(\frac{-Z_0 - \hat{\mu}T}{\sigma\sqrt{T}}\right) + e^{-\frac{2Z_0\hat{\mu}}{\sigma^2}} N\left(\frac{-Z_0 + \hat{\mu}T}{\sigma\sqrt{T}}\right) \\ &= N\left(\frac{\ln\left(\frac{B_0^*}{A_0}\right) - (\mu - g - \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}}\right) + \left(\frac{A_0}{B_0}\right)^{-\frac{2(\mu - g - \frac{1}{2}\sigma^2)}{\sigma^2}} N\left(\frac{\ln\left(\frac{B_0}{A_0}\right) + (\mu - g - \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}}\right) \end{aligned}$$

B Derivation of the expected payoff given default

The expected payoff given default is given by

$$\frac{E\left[(\eta \wedge 1)\, L_0 e^{g\tau} e^{r(T-\tau)} \mathbf{1}_{\{\tau \leq T\}}\right]}{P\{\tau \leq T\}}$$

The denominator is already calculated in Appendix A, we just need to calculate the numerator. The numerator is given as follows:

$$\begin{aligned}
& E\left[(\eta \wedge 1)\, L_0 e^{g\tau} e^{r(T-\tau)} \mathbf{1}_{\{\tau \leq T\}}\right] \\
= \; & (\eta \wedge 1)\, L_0 e^{g\,T} \int_0^T \frac{1}{\sqrt{2\pi}} \frac{Z_0}{\sigma\tau^{3/2}} \exp\{-(r-g)\tau\} \exp\left\{-\frac{1}{2}\left(\frac{Z_0+\hat{\mu}\tau}{\sigma\sqrt{\tau}}\right)^2\right\} d\tau \\
= \; & (\eta \wedge 1)\, L_0 e^{g\,T} \int_0^T \frac{Z_0}{\sigma\tau^{3/2}} \exp\{-(r-g)\tau\} n\left(\frac{Z_0}{\sigma\sqrt{\tau}} + \frac{\hat{\mu}}{\sigma}\sqrt{\tau}\right) d\tau \\
= \; & (\eta \wedge 1)\, L_0 e^{g\,T} \int_0^T \frac{Z_0}{\sigma\tau^{3/2}} \exp\left\{-\frac{Z_0}{\sigma}\frac{\hat{\mu}}{\sigma} + \frac{Z_0}{\sigma}\sqrt{\frac{\hat{\mu}^2}{\sigma^2} + 2(r-g)}\right\} n\left(\frac{Z_0}{\sigma\sqrt{\tau}} + \sqrt{\frac{\hat{\mu}^2}{\sigma^2} + 2(r-g)}\sqrt{\tau}\right) d\tau \\
= \; & (\eta \wedge 1)\, L_0 e^{g\,T} \left(\frac{A_0}{\eta L_0}\right)^{-\frac{\hat{\mu}}{\sigma^2} + \frac{1}{\sigma}\sqrt{\frac{\hat{\mu}^2}{\sigma^2} + 2(r-g)}} \underbrace{\int_0^T \frac{1}{2\pi} \frac{Z_0}{\sigma\tau^{3/2}} \exp\left(-\frac{(Z_0 + \sqrt{\hat{\mu}^2 + 2(r-g)\sigma^2}\tau)^2}{2\,\sigma^2\,\tau}\right) d\tau}_{\text{density of the first passage time at} \sqrt{\hat{\mu}^2 + 2(r-g)\sigma^2}} \\
= \; & (\eta \wedge 1)\, L_0 e^{g\,T} \left(\frac{A_0}{\eta L_0}\right)^{-\frac{\hat{\mu}}{\sigma^2} + \frac{1}{\sigma}\sqrt{\frac{\hat{\mu}^2}{\sigma^2} + 2(r-g)}} \left\{ N\left(\frac{\ln(\frac{\eta L_0}{A_0}) - \sqrt{(\hat{\mu})^2 + 2(r-g)\sigma^2}T}{\sigma\sqrt{T}}\right)\right. \\
& \left. + \left(\frac{A_0}{\eta L_0}\right)^{\frac{-2}{\sigma^2}\sqrt{(\hat{\mu})^2 + 2(r-g)\sigma^2}} N\left(\frac{\ln(\frac{\eta L_0}{A_0}) + \sqrt{(\hat{\mu})^2 + 2(r-g)\sigma^2}T}{\sigma\sqrt{T}}\right)\right\} \\
= \; & (\eta \wedge 1)\, L_0 e^{g\,T} \left\{ \left(\frac{\eta L_0}{A_0}\right)^{\frac{\hat{\mu}}{\sigma^2}} \left(\frac{A_0}{\eta L_0}\right)^{\frac{\sqrt{(\hat{\mu})^2 + 2(r-g)\sigma^2}}{\sigma^2}} N\left(\frac{\ln(\frac{\eta L_0}{A_0}) - \sqrt{(\hat{\mu})^2 + 2(r-g)\sigma^2}T}{\sigma\sqrt{T}}\right)\right. \\
& \left. + \left(\frac{\eta L_0}{A_0}\right)^{\frac{\hat{\mu} + \sqrt{\hat{\mu}^2 + 2(r-g)\sigma^2}}{\sigma^2}} N\left(\frac{\ln(\frac{\eta L_0}{A_0}) + \sqrt{(\hat{\mu})^2 + 2(r-g)\sigma^2}T}{\sigma\sqrt{T}}\right)\right\}
\end{aligned}$$

C Derivation of default probability in Parisian framework

The default probability in the case of standard Parisian option is given by

$$\begin{aligned}
P^{\varepsilon} &= P\left(T_B^- = \inf\left\{t > 0 \middle| \left(t - g_{B,t}^A\right) \mathbb{1}_{\{A_t < B_t\}} > d\right\} \leq T\right) \\
&= e^{-\frac{1}{2}m^2 T}\left(\int_{-\infty}^b h_2(T,y)e^{my}\,d\,y + \int_b^\infty h_1(T,y)e^{my}\,d\,y\right)
\end{aligned}$$

with $m = \frac{1}{\sigma}\left(\mu - g - \frac{1}{2}\sigma^2\right)$. $h_1(T, y)$ and $h_2(T, y)$ are uniquely determined by inverting the corresponding Laplace transforms which are given by

$$\begin{aligned}
\hat{h}_1(\lambda, y) &= \frac{e^{(2b-y)\sqrt{2\lambda}}\psi\left(-\sqrt{2\lambda d}\right)}{\sqrt{2\lambda}\psi\left(\sqrt{2\lambda d}\right)} \\
\hat{h}_2(\lambda, y) &= \frac{e^{y\sqrt{2\lambda}}}{\sqrt{2\lambda}\psi\left(\sqrt{2\lambda d}\right)} + \frac{\sqrt{2\lambda d}e^{\lambda d}}{\psi\left(\sqrt{2\lambda d}\right)}\left(e^{y\sqrt{2\lambda}}\left(N\left(-\sqrt{2\lambda d} - \frac{y-b}{\sqrt{d}}\right) - N\left(-\sqrt{2\lambda d}\right)\right)\right. \\
&\quad \left. -e^{(2b-y)\sqrt{2\lambda}}N\left(-\sqrt{2\lambda d} + \frac{y-b}{\sqrt{d}}\right)\right).
\end{aligned}$$

with

$$\begin{aligned}
b &= \frac{1}{\sigma}\ln\left(\frac{B_0}{A_0}\right) = \frac{1}{\sigma}\ln\left(\frac{\eta L_0}{A_0}\right) = \frac{1}{\sigma}\ln\left(\eta\alpha\right) < 0 \\
\psi(z) &= \int_0^\infty x\exp\left\{-\frac{x^2}{2} + zx\right\}dx = 1 + z\sqrt{2\pi}e^{\frac{z^2}{2}}N(z),
\end{aligned}$$

and λ the parameter of Laplace transform. The default probability in the case of cumulative Parisian option is determined by

$$\begin{aligned}
P^\varepsilon &= P(\tau \le T) = P\left(\frac{1}{T}\int_0^T \mathbb{1}_{\{W_u + m\,u \le b\}}du \ge \frac{d}{T}\right) \\
&= P\left(\frac{1}{T}\int_0^T \mathbb{1}_{\{W_u - m\,u \le -b\}}du \le 1 - \frac{d}{T}\right) \\
&= 2\int_0^{1-\frac{d}{T}}\left\{\left[\frac{N\left(-m\sqrt{T}\sqrt{1-u}\right)}{\sqrt{1-u}} - m\sqrt{T}N\left(-m\sqrt{T}\sqrt{1-u}\right)\right]\right. \\
&\quad \left.\cdot\left[\frac{1}{\sqrt{u}}N\left(\frac{(-b)/\sqrt{T} + m\sqrt{T}u}{\sqrt{u}}\right) + m\sqrt{T}e^{2mb}N\left(\frac{b/\sqrt{T} + m\sqrt{T}u}{\sqrt{u}}\right)\right]\right\}du,
\end{aligned}$$

where $N(\cdot)$ is the density function of the standard normal distribution. In the above derivation, Equation (12) of Takács [1996] is applied.

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