Robust and Efficient Fitting of Claim Severity Distributions

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44th Actuarial Research Conference

Madison, WI, July 30–August 1, 2009

[a] In collaboration with Jones and Zitikis (University of Western Ontario)
[b] Supported by a grant from the Actuarial Foundation, SOA, and CAS

Outline

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2. **Method of Trimmed Moments**
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   - Examples
   - Simulations

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   - Real-Data Examples
   - Concluding Remarks

1. Introduction

Preliminaries

- **Claim Severity Distributions**
  - ▷ STATISTICAL OBJECTIVE:
    - + Accurate model fit
  - ▷ ACTUARIAL OBJECTIVES:
    - + Risk evaluations
    - + Ratemaking
    - + Reserve calculations

- **Standard Estimation & Fitting Techniques**
  - ▷ EMPIRICAL NONPARAMETRIC
    - + Simple approach
    - + Weak assumptions

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  - ▷ Parametric
    - + Efficiency
    - + Smoothness
    - + Stretchability beyond the range of observed data
    - + Special distributional features (e.g., mode at 0)

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  - ▷ PARAMETRIC
    - \+ Efficiency
    - \+ Smoothness
    - \+ Stretchability beyond the range of observed data
    - \+ Special distributional features (e.g., mode at 0)
    - – Strong assumptions
    - – Outliers (e.g., loss that receives an extensive media attention)

Motivation

- ▷ NOT ROBUST: maximum likelihood, method-of-moments
- ▷ ROBUST: M-, L-, R-statistics

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- ▷ L ($linear$ combinations of order statistics)
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  - + Computer friendly
  - + Transparent

2. Method of Trimmed Moments

Definition

- **Assumptions & Notation**
  - ▷ DATA: $X_1, \ldots, X_n$ *i.i.d.* with cdf F
  - ▷ CDF:
    - \+ F is *continuous*
    - \+ F depends on $\theta_1, \ldots, \theta_k$ (*unknown* parameters)
  - ▷ ORDERED DATA: $X_{1:n} \leq \cdots \leq X_{n:n}$

- **Three-Step Procedure**

1. SAMPLE TRIMMED MOMENTS:

$$\widehat{\mu}_j = \frac{1}{n - m_n(j) - m_n^*(j)} \sum_{i=m_n(j)+1}^{n-m_n^*(j)} h_j(X_{i:n})$$

$j = 1, \ldots, k,$ with $m_n(j)/n \approx a_j,$ $m_n^*(j)/n \approx b_j$ chosen trimming proportions, h_j chosen function.

- **Three-Step Procedure**

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$j = 1, \ldots, k$, with $m_n(j)/n \approx a_j$, $m_n^*(j)/n \approx b_j$ chosen trimming proportions, h_j chosen function.

2. Population Trimmed Moments:

$$\mu_j := \mu_j(\theta_1, \ldots, \theta_k) = \frac{1}{1 - a_j - b_j} \int_{a_j}^{1-b_j} h_j(F^{-1}(u))\, \mathrm{d}u$$

$j = 1, \ldots, k$.

3. MATCH & SOLVE:

$$\left\{ \begin{array}{rcl} \mu_1(\theta_1, \ldots, \theta_k) & = & \widehat{\mu}_1, \\ & \vdots & \\ \mu_k(\theta_1, \ldots, \theta_k) & = & \widehat{\mu}_k. \end{array} \right.$$

3. MATCH & SOLVE:

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- **MTM estimators of** $\theta_1, \ldots, \theta_k$

$$\widehat{\theta}_1 = g_1(\widehat{\mu}_1, \ldots, \widehat{\mu}_k), \quad \ldots\ldots, \widehat{\theta}_k = g_k(\widehat{\mu}_1, \ldots, \widehat{\mu}_k).$$

Asymptotic Properties

$$\left(\widehat{\theta}_1, \ldots, \widehat{\theta}_k\right) \quad \text{is} \quad \mathcal{AN}\left((\theta_1, \ldots, \theta_k),\, n^{-1}\,\mathbf{D}\boldsymbol{\Sigma}\mathbf{D}'\right)$$

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$$\left(\widehat{\theta}_1, \ldots, \widehat{\theta}_k\right) \quad \text{is} \quad \mathcal{AN}\Big((\theta_1, \ldots, \theta_k),\, n^{-1}\, \mathbf{D}\boldsymbol{\Sigma}\mathbf{D}' \Big)$$

where $\mathbf{D}_{k\times k}$ with $d_{ij} = \frac{\partial g_i}{\partial \widehat{\mu}_j}\Big|_{(\mu_1,\ldots,\mu_k)}$ and $\boldsymbol{\Sigma}_{k\times k}$ with

$$\begin{aligned} \sigma_{ij}^2 &= \frac{1}{(1-a_i-b_i)(1-a_j-b_j)} \\ &\times \int_{a_i}^{1-b_i} \int_{a_j}^{1-b_j} \big(\min\{u,v\} - uv \big)\, \mathrm{d}h_j\big(F^{-1}(v)\big)\, \mathrm{d}h_i\big(F^{-1}(u)\big) \end{aligned}$$

Examples

- **Location-Scale Families**

  ▷ CDF, QF:

$$F(x) = F_0\left(\frac{x-\theta}{\sigma}\right), \quad -\infty < x < \infty,$$

$$F^{-1}(u) = \theta + \sigma F_0^{-1}(u), \quad 0 < u < 1.$$

where $\theta \in \mathcal{R}$, $\sigma > 0$, and F_0 is the standard version of F.

  ▷ Functions h:

$$h_1(t) = t, \qquad h_2(t) = t^2.$$

▷ Sample TMs:

$$\widehat{\mu}_j = \frac{1}{n - m_n - m_n^*} \sum_{i=m_n+1}^{n-m_n^*} X_{i:n}^j, \qquad j = 1, 2$$

▷ Population TMs:

$$\mu_1 = \frac{1}{1-a-b} \int_a^{1-b} F^{-1}(u)\, \mathrm{d}u = \theta + \sigma \times c_1$$

$$\mu_2 = \frac{1}{1-a-b} \int_a^{1-b} \left[F^{-1}(u)\right]^2 \mathrm{d}u = \theta^2 + 2\theta\sigma \times c_1 + \sigma^2 \times c_2$$

▷ MTM of (θ, σ):

$$\begin{cases} \widehat{\theta}_{\text{MTM}} &= \widehat{\mu}_1 - c_1\, \widehat{\sigma}_{\text{MTM}} \\ \widehat{\sigma}_{\text{MTM}} &= \sqrt{(\widehat{\mu}_2 - \widehat{\mu}_1^2)/(c_2 - c_1^2)} \end{cases}$$

▷ ASYMPTOTICS:

$$\left(\widehat{\theta}_{\text{MTM}},\, \widehat{\sigma}_{\text{MTM}}\right) \text{ is } \mathcal{AN}\left((\theta, \sigma),\, \frac{\sigma^2}{n}\,\mathbf{S}\right)$$

▷ EXAMPLES of F_0 and $\log F_0$:

Cauchy, Gumbel, Laplace, Logistic, Normal, Student's t; and log-Cauchy, Weibull, log-Laplace, loglogistic, lognormal, log-t.

- **Lognormal Model**

  ▷ CDF, QF:

$$F(x) = \Phi\left(\frac{\log(x - x_0) - \theta}{\sigma}\right)$$

$$\log(F^{-1}(u) - x_0) = \theta + \sigma\,\Phi^{-1}(u)$$

$\theta \in \mathcal{R}$, $\sigma > 0$, $x > x_0$ (*known* deductible), $0 < u < 1$, and Φ, Φ^{-1} are CDF, QF of $N(0,1)$.

  ▷ Functions h:

$$h_1(t) = \log(t - x_0), \qquad h_2(t) = \log^2(t - x_0)$$

$$\left(\widehat{\theta}_{\text{MLE}},\ \widehat{\sigma}_{\text{MLE}}\right) \quad \text{is} \quad \mathcal{AN}\left((\theta,\sigma),\ \frac{\sigma^2}{n}\,\mathbf{S_0}\right)$$

TABLE 1: $\text{ARE}((\widehat{\theta}_{\text{MTM}}, \widehat{\sigma}_{\text{MTM}}),\ (\widehat{\theta}_{\text{MLE}}, \widehat{\sigma}_{\text{MLE}})) = \sqrt{|\mathbf{S_0}|/|\mathbf{S}|}$.

	b				
a	0	0.05	0.15	0.49	0.70
0	1	.932	.821	.502	.312
0.05		.872	.771	.470	.286
0.15			.676	.390	.208
0.49				.074	–
0.70					–

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- **Pareto Model**

  ▷ CDF, QF:

$$F(x) = 1 - \left(\frac{x}{x_0}\right)^{-\alpha},$$

$$F^{-1}(u) = x_0(1-u)^{-1/\alpha}$$

$\alpha > 0$, $x > x_0$ (*known* deductible), $0 < u < 1$.

  ▷ FUNCTION h_1:

$$h_1(t) = \log t$$

▷ MTM of α:

$$\widehat{\alpha}_{\mathrm{MTM}} = \frac{const_1}{\widehat{\mu}_1}$$

▷ ASYMPTOTICS:

$$\widehat{\alpha}_{\mathrm{MTM}} \quad \text{is} \quad \mathcal{AN}\left(\alpha,\, \frac{\alpha^2}{n}\, Const_1\right)$$

▷ COMPARISON with MLE:

$$\widehat{\alpha}_{\mathrm{MLE}} = \frac{n}{\sum_{i=1}^{n} \log(X_i/x_0)} \quad \text{is} \quad \mathcal{AN}\left(\alpha,\, \frac{\alpha^2}{n}\right)$$

Table 2: $\text{ARE}(\widehat{\alpha}_{\text{MTM}}, \widehat{\alpha}_{\text{MLE}}) = 1/Const_1$.

	b						
a	0	0.05	0.10	0.15	0.25	0.49	0.70
0	1	.918	.847	.783	.666	.423	.238
0.05		.918	.848	.783	.667	.425	.242
0.10			.848	.785	.669	.430	.250
0.15				.787	.672	.437	.261
0.25					.679	.452	.285
0.49						.487	–
0.70							–

TABLE 2: $\text{ARE}(\widehat{\alpha}_{\text{MTM}},\, \widehat{\alpha}_{\text{MLE}}) = 1/Const_1$.

a	b: 0	0.05	0.10	0.15	0.25	0.49	0.70
0	1						
0.05	1.00	.918					
0.10	1.00	.918	.848				
0.15	.999	.919	.850	.787			
0.25	.995	.918	.851	.790	.679		
0.49	.958	.897	.839	.786	.688	.487	
0.70	.857	.824	.781	.738	.659	–	–

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Simulations

- **Study Design**
  - ▷ NUMBER OF SIMULATED SAMPLES: $M = 100,000$
  - ▷ SIZES OF SAMPLES: $n = 50,\ 100,\ 250,\ 500$

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  - ▷ SELECTED MODELS:
    - + Pareto($x_0 = 1,\ \alpha = 0.50$)
    - + Lognormal($x_0 = 1,\ \theta = 5,\ \sigma = 3$)

Simulations

- **Study Design**
  - ▷ NUMBER OF SIMULATED SAMPLES: $M = 100,000$
  - ▷ SIZES OF SAMPLES: $n = 50,\ 100,\ 250,\ 500$
  - ▷ SELECTED MODELS:
    - + Pareto($x_0 = 1,\ \alpha = 0.50$)
    - + Lognormal($x_0 = 1,\ \theta = 5,\ \sigma = 3$)
  - ▷ METHODS OF ESTIMATION: MLE, MTM
  - ▷ REPORTING: standardized MEAN, RE

TABLE 3: Pareto($x_0 = 1,\ \alpha = 0.50$) model.

Statistic	*Estimator* a	b	*Sample Size* 50	100	250	500	∞
MEAN/α	0	0	1.02				1
	0.05	0.05	0.99				1
	0.10	0.10	1.01				1
	0.25	0.25	1.01				1
	0.49	0.49	1.03				1
	0.10	0.70	1.04				1
	0.25	0.00	1.03				1

NOTE: Standard errors for all entries $\leq .001$

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MEAN/α	0	0	1.02	1.01			1
	0.05	0.05	0.99	1.01			1
	0.10	0.10	1.01	1.01			1
	0.25	0.25	1.01	1.01			1
	0.49	0.49	1.03	1.01			1
	0.10	0.70	1.04	1.02			1
	0.25	0.00	1.03	1.01			1

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	0.05	0.05	0.99	1.01	1.00		1
	0.10	0.10	1.01	1.01	1.00		1
	0.25	0.25	1.01	1.01	1.00		1
	0.49	0.49	1.03	1.01	1.01		1
	0.10	0.70	1.04	1.02	1.01		1
	0.25	0.00	1.03	1.01	1.01		1

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MEAN/α	0	0	1.02	1.01	1.00	1.00	1
	0.05	0.05	0.99	1.01	1.00	1.00	1
	0.10	0.10	1.01	1.01	1.00	1.00	1
	0.25	0.25	1.01	1.01	1.00	1.00	1
	0.49	0.49	1.03	1.01	1.01	1.00	1
	0.10	0.70	1.04	1.02	1.01	1.00	1
	0.25	0.00	1.03	1.01	1.01	1.00	1

NOTE: Standard errors for all entries $\leq .001$

TABLE 4: Pareto($x_0 = 1,\ \alpha = 0.50$) model.

Statistic	*Estimator* a	b	*Sample Size* 50	100	250	500	∞
RE	0	0	0.92				1
	0.05	0.05	0.90				0.918
	0.10	0.10	0.80				0.848
	0.25	0.25	0.65				0.679
	0.49	0.49	0.43				0.487
	0.10	0.70	0.21				0.250
	0.25	0.00	0.87				0.995

NOTE: Standard errors for all entries $\leq .006$

TABLE 4: Pareto($x_0 = 1,\ \alpha = 0.50$) model.

Statistic	*Estimator* a	b	*Sample Size* 50	100	250	500	∞
RE	0	0	0.92	0.96			1
	0.05	0.05	0.90	0.92			0.918
	0.10	0.10	0.80	0.83			0.848
	0.25	0.25	0.65	0.65			0.679
	0.49	0.49	0.43	0.45			0.487
	0.10	0.70	0.21	0.23			0.250
	0.25	0.00	0.87	0.95			0.995

NOTE: Standard errors for all entries $\leq .006$

TABLE 4: Pareto($x_0 = 1,\ \alpha = 0.50$) model.

Statistic	*Estimator* a	b	*Sample Size* 50	100	250	500	∞
RE	0	0	0.92	0.96	0.98		1
	0.05	0.05	0.90	0.92	0.92		0.918
	0.10	0.10	0.80	0.83	0.84		0.848
	0.25	0.25	0.65	0.65	0.68		0.679
	0.49	0.49	0.43	0.45	0.47		0.487
	0.10	0.70	0.21	0.23	0.24		0.250
	0.25	0.00	0.87	0.95	0.97		0.995

NOTE: Standard errors for all entries $\leq .006$

Table 4: Pareto($x_0 = 1,\ \alpha = 0.50$) model.

Statistic	*Estimator* a	b	*Sample Size* 50	100	250	500	∞
RE	0	0	0.92	0.96	0.98	1.00	1
	0.05	0.05	0.90	0.92	0.92	0.92	0.918
	0.10	0.10	0.80	0.83	0.84	0.85	0.848
	0.25	0.25	0.65	0.65	0.68	0.68	0.679
	0.49	0.49	0.43	0.45	0.47	0.48	0.487
	0.10	0.70	0.21	0.23	0.24	0.25	0.250
	0.25	0.00	0.87	0.95	0.97	0.99	0.995

Note: Standard errors for all entries $\leq .006$

TABLE 5: Lognormal($x_0 = 1$, $\theta = 5$, $\sigma = 3$) model.

Statistic	*Estimator* a	b	*Sample Size* 50	100	250	500	∞
RE	0	0	0.99				1
	0.05	0.05	0.82				0.872
	0.10	0.10	0.77				0.769
	0.25	0.25	0.48				0.507
	0.49	0.49	0.04				0.074
	0.10	0.70	0.24				0.248
	0.25	0.00	0.73				0.722

NOTE: Standard errors for all entries $\leq .003$

TABLE 5: Lognormal($x_0 = 1$, $\theta = 5$, $\sigma = 3$) model.

Statistic	*Estimator* a	b	*Sample Size* 50	100	250	500	∞
RE	0	0	0.99	1.00			1
	0.05	0.05	0.82	0.87			0.872
	0.10	0.10	0.77	0.77			0.769
	0.25	0.25	0.48	0.50			0.507
	0.49	0.49	0.04	0.06			0.074
	0.10	0.70	0.24	0.25			0.248
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Statistic	*Estimator* a	b	*Sample Size* 50	100	250	500	∞
RE	0	0	0.99	1.00	1.00	1.00	1
	0.05	0.05	0.82	0.87	0.87	0.87	0.872
	0.10	0.10	0.77	0.77	0.77	0.77	0.769
	0.25	0.25	0.48	0.50	0.50	0.51	0.507
	0.49	0.49	0.04	0.06	0.07	0.07	0.074
	0.10	0.70	0.24	0.25	0.25	0.25	0.248
	0.25	0.00	0.73	0.72	0.72	0.72	0.722

NOTE: Standard errors for all entries $\leq .003$

3. Illustrations and Conclusions

Real-Data Examples: Hurricane Damages

- **Data**
  - ▷ Top 30 damaging hurricanes in the United States: 1925–1995.
  - ▷ Normalized to 1995 dollars by inflation, personal property increases, coastal county population changes.
  - ▷ Published by Pielke and Landsea (1998) in *Weather and Forecasting*.
- **Objectives**
  - ▷ STATISTICAL: Model fitting
  - ▷ ACTUARIAL: Ratemaking

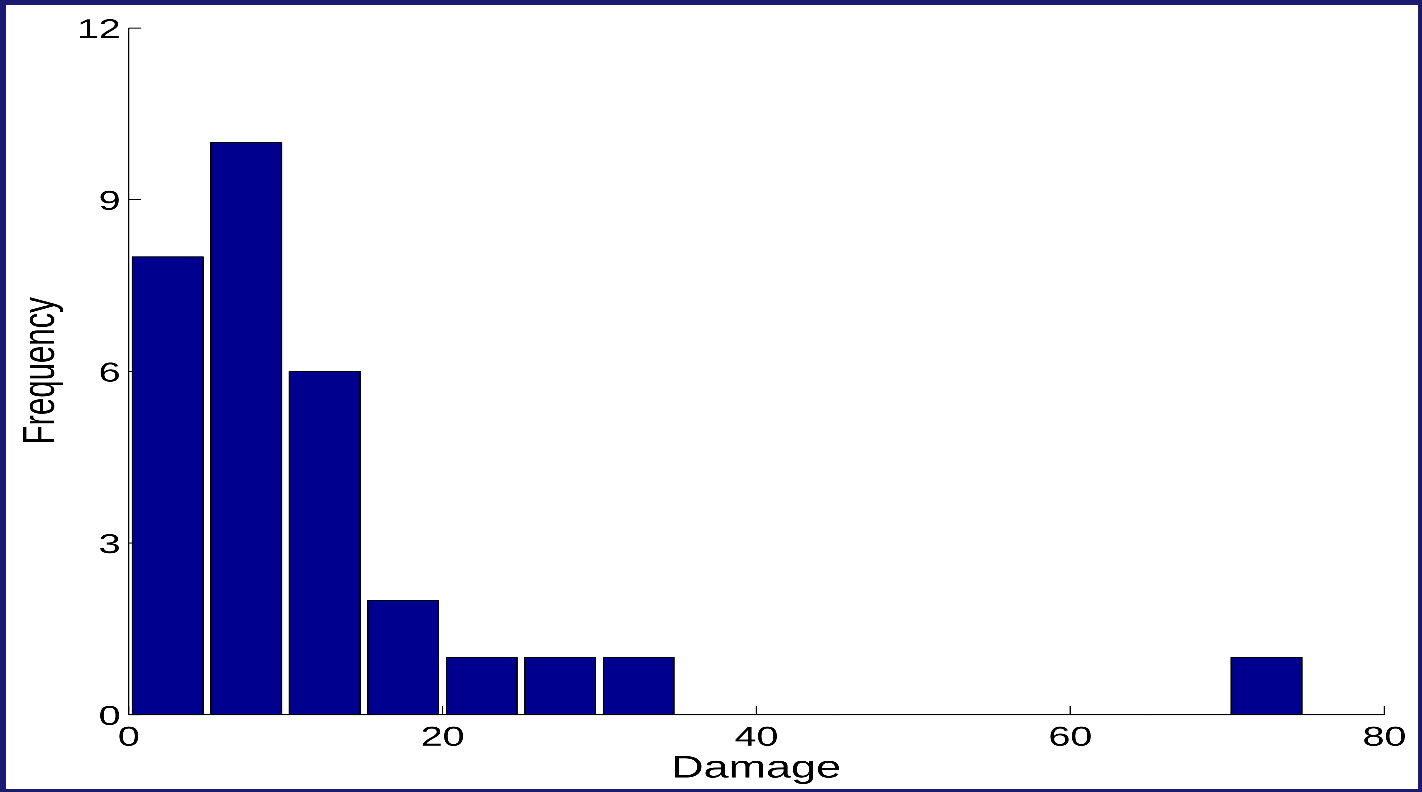


FIGURE 1: Histogram of the top 30 damaging hurricanes.

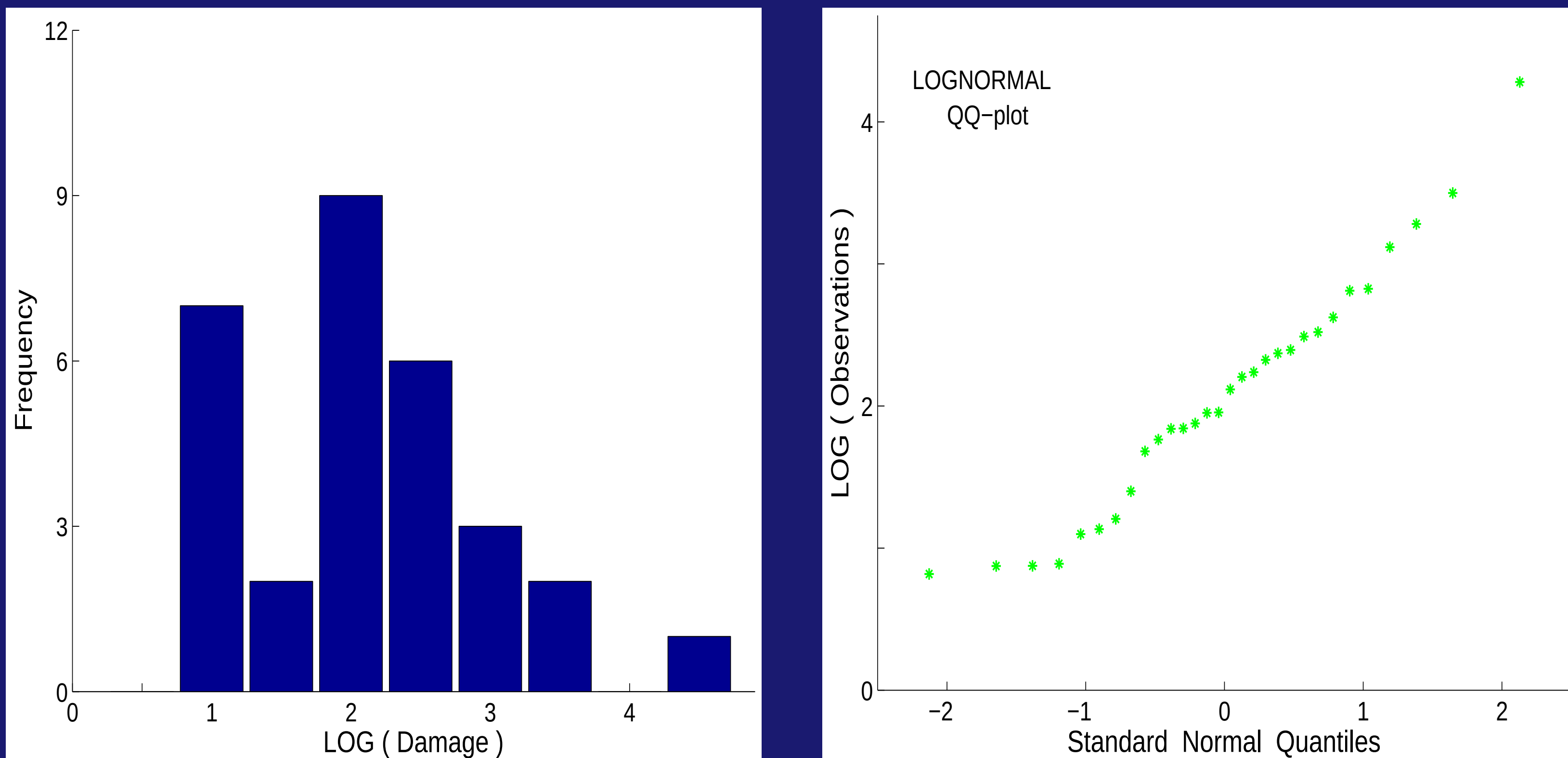
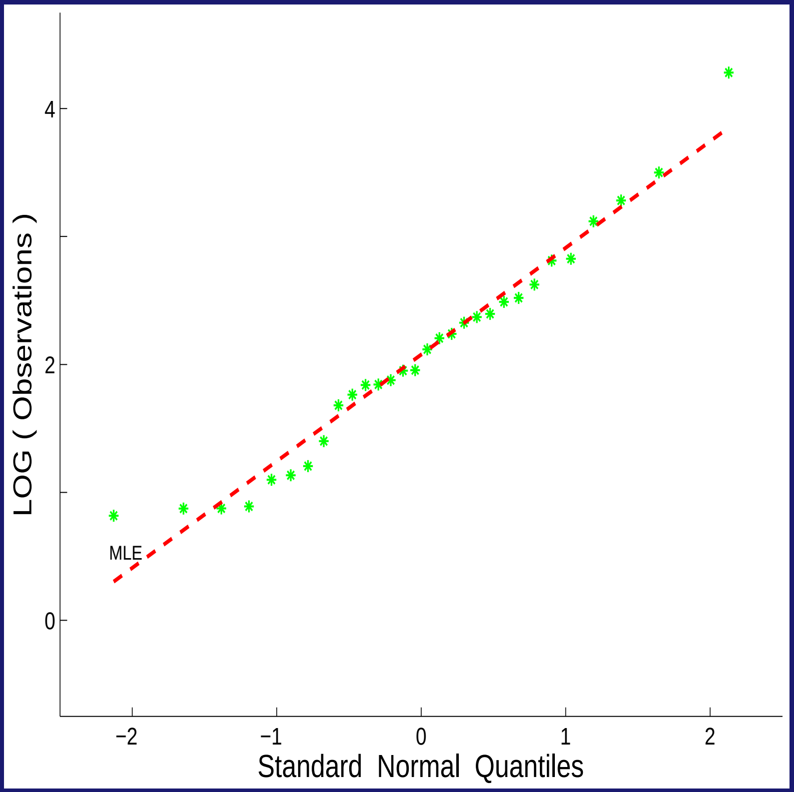
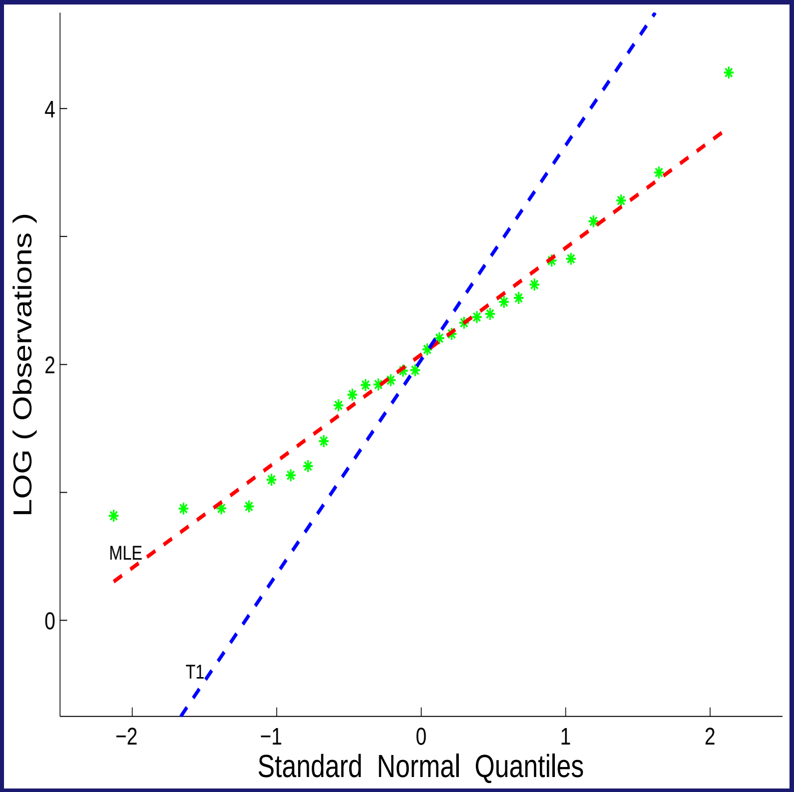


FIGURE 2: Preliminary diagnostics for the hurricane data.



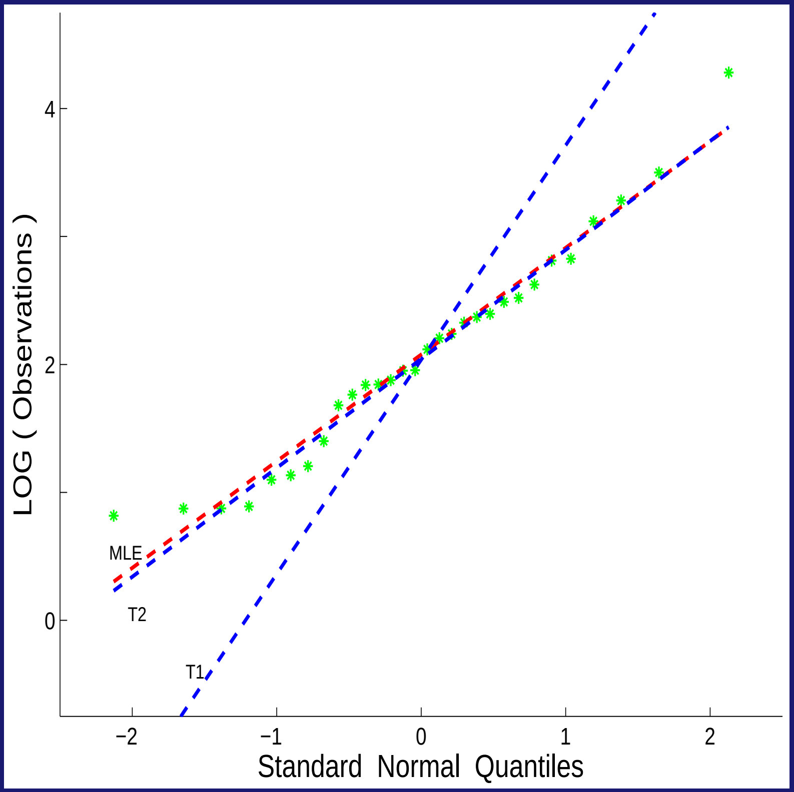
Estimator	$\widehat{\theta}$	$\widehat{\sigma}$	*Fit*
MLE	2.077	0.834	0.104

FIGURE 3: Lognormal fits to the hurricane data.



Estimator	$\widehat{\theta}$	$\widehat{\sigma}$	*Fit*
MLE	2.077	0.834	0.104
$T1\left(\frac{14}{30}, \frac{14}{30}\right)$	2.037	1.675	0.662

FIGURE 3: Lognormal fits to the hurricane data.



Estimator	$\widehat{\theta}$	$\widehat{\sigma}$	*Fit*
MLE	2.077	0.834	0.104
$T1\left(\frac{14}{30}, \frac{14}{30}\right)$	2.037	1.675	0.662
$T2\left(\frac{1}{30}, \frac{1}{30}\right)$	2.043	0.852	0.101

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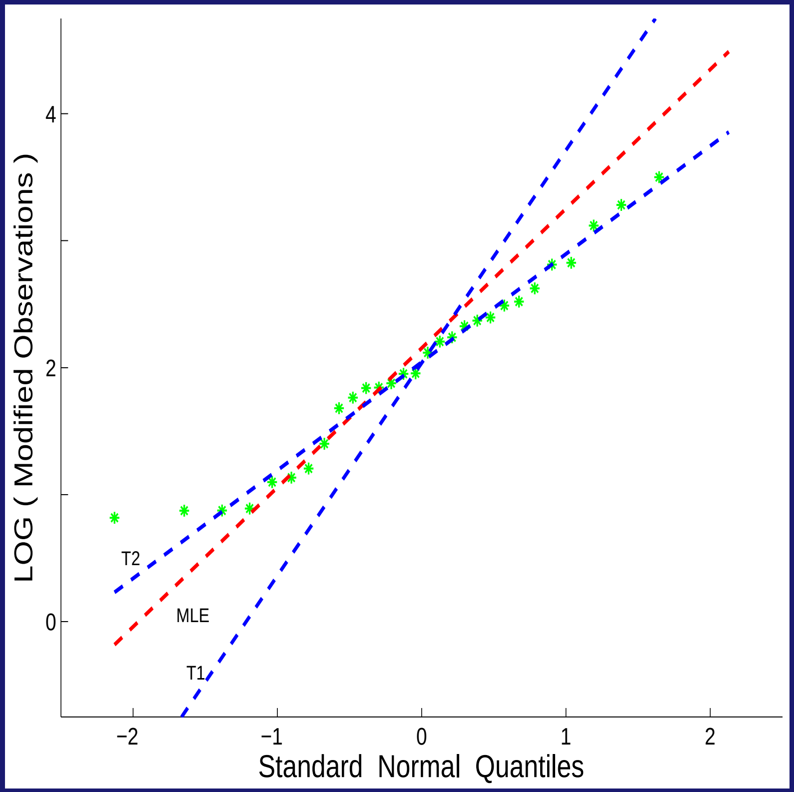


Estimator	$\widehat{\theta}$	$\widehat{\sigma}$	*Fit*
MLE	2.154	1.098	0.293
T1$\left(\frac{14}{30}, \frac{14}{30}\right)$	2.037	1.675	0.651
T2$\left(\frac{1}{30}, \frac{1}{30}\right)$	2.043	0.852	0.178

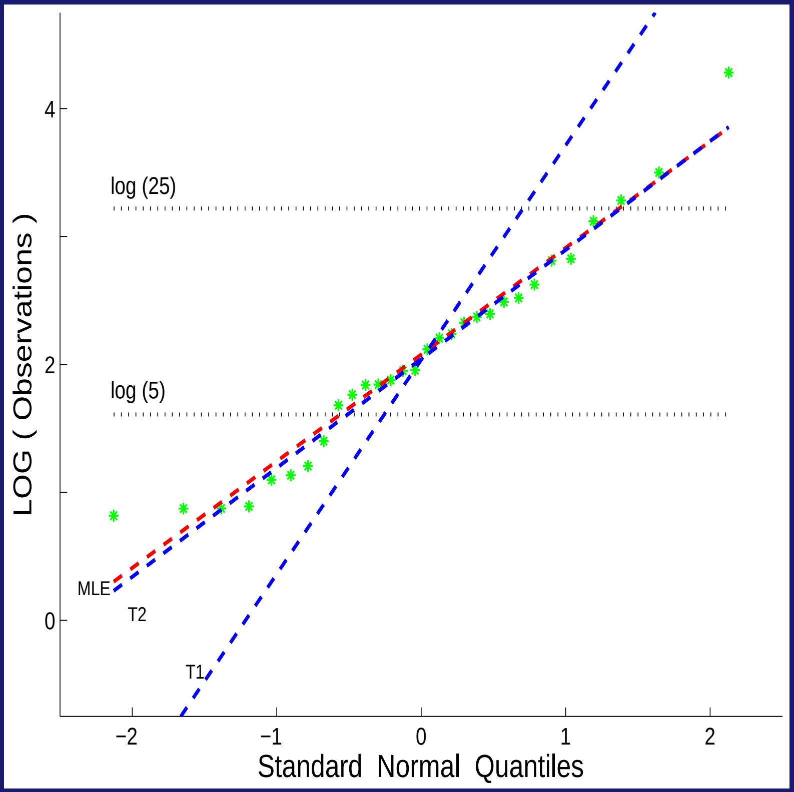
FIGURE 3: Lognormal fits to the *modified* hurricane data. (*Largest observation* 72.303 *is replaced with* 723.03)

- **Insurance Contract**

  Insurance benefit equal to the amount by which a hurricane's damage exceeds 5 (billion) with a maximum benefit of 20.

- **Net Premium**

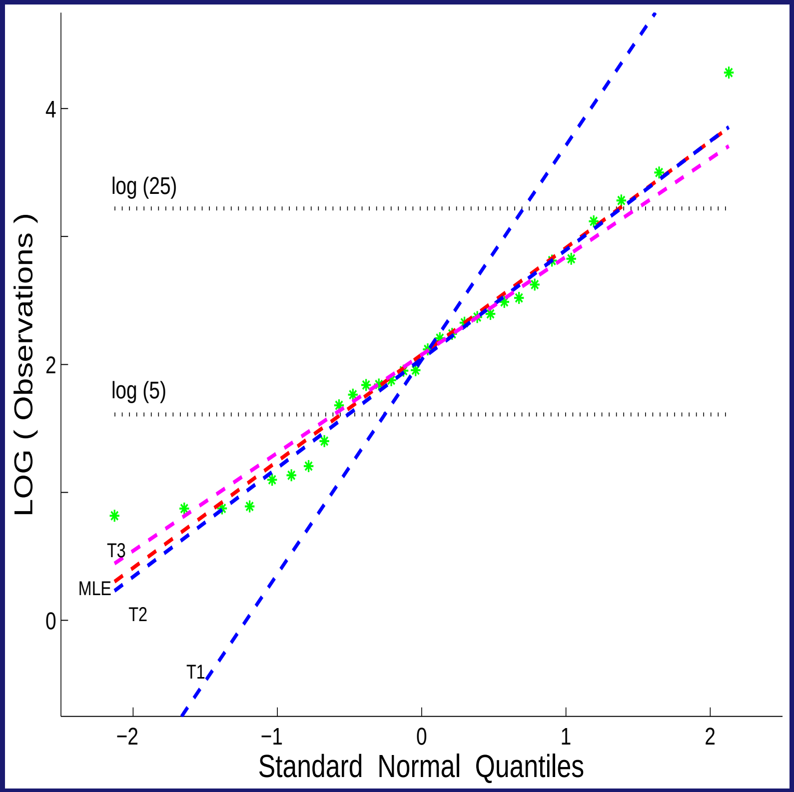
$$\text{PREMIUM} = \int_{5}^{25} (x-5)\, \mathrm{d}F(x) + 20[1 - F(25)]$$



Estimator	$\widehat{\theta}$	$\widehat{\sigma}$	*R-Fit*
MLE	2.077	0.834	0.054
T1 $\left(\frac{14}{30}, \frac{14}{30}\right)$	2.037	1.675	0.413
T2 $\left(\frac{1}{30}, \frac{1}{30}\right)$	2.043	0.852	0.057

PREMIUM (EMP)	5.42
PREMIUM (MLE)	5.60
PREMIUM (T1)	7.35
PREMIUM (T2)	5.44

FIGURE 4: Lognormal fits to the hurricane data.



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T2$\left(\frac{1}{30}, \frac{1}{30}\right)$	2.043	0.852	0.057
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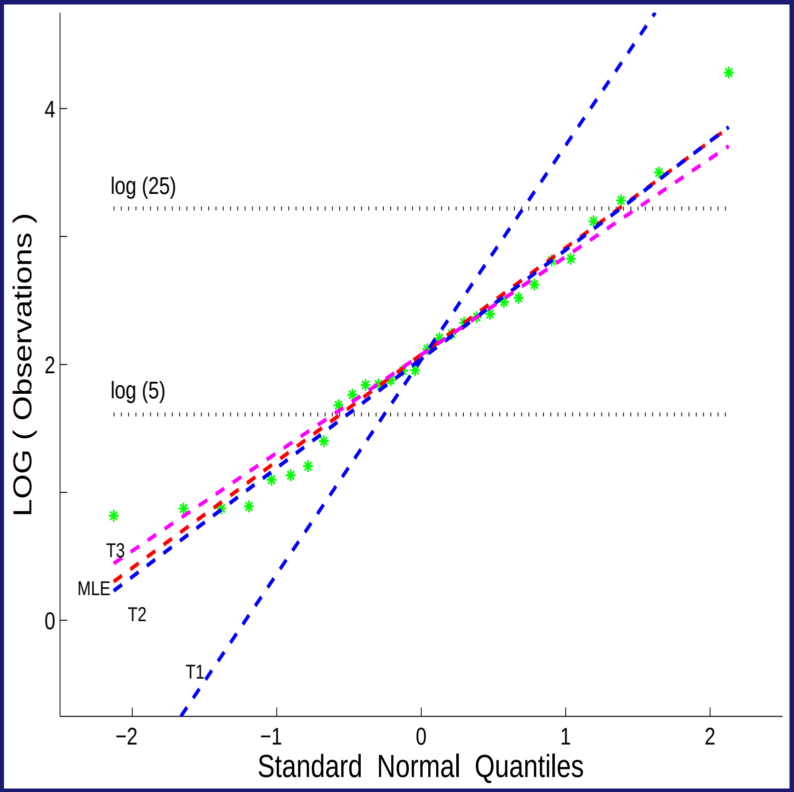

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T3$\left(\frac{8}{30}, \frac{3}{30}\right)$	2.075	0.766	0.042

PREMIUM (EMP)	5.42 (3.11; 7.72)
PREMIUM (MLE)	5.60 (3.37; 7.84)
PREMIUM (T1)	7.35 (2.53; 12.16)
PREMIUM (T2)	5.44 (3.17; 7.71)
PREMIUM (T3)	5.34 (3.07; 7.61)

FIGURE 4: Lognormal fits to the hurricane data.

Real-Data Examples: Norwegian Fire Claims

- **Data**
  - ▷ Total damage done by 827 fires in Norway for the year 1988.
  - ▷ All claims exceed 500 thousand Norwegian krones (NOK); the *deductible* is 500,000 NOK.
  - ▷ Published by Beirlant, Teugels, and Vynckier (1996).

- **Objectives**
  - ▷ Statistical: Model fitting
  - ▷ Actuarial: Risk evaluations

Claim Sizes (in 1000s)			*Frequency*
500	$\longrightarrow$	$1,000$	341
$1,000$	$\longrightarrow$	$2,000$	271
$2,000$	$\longrightarrow$	$5,000$	140
$5,000$	$\longrightarrow$	$10,000$	43
$10,000$	$\longrightarrow$	$50,000$	28
$50,000$	$+$		4

Top 4 claims: $61,937$; $84,464$; $150,597$; $465,365$.

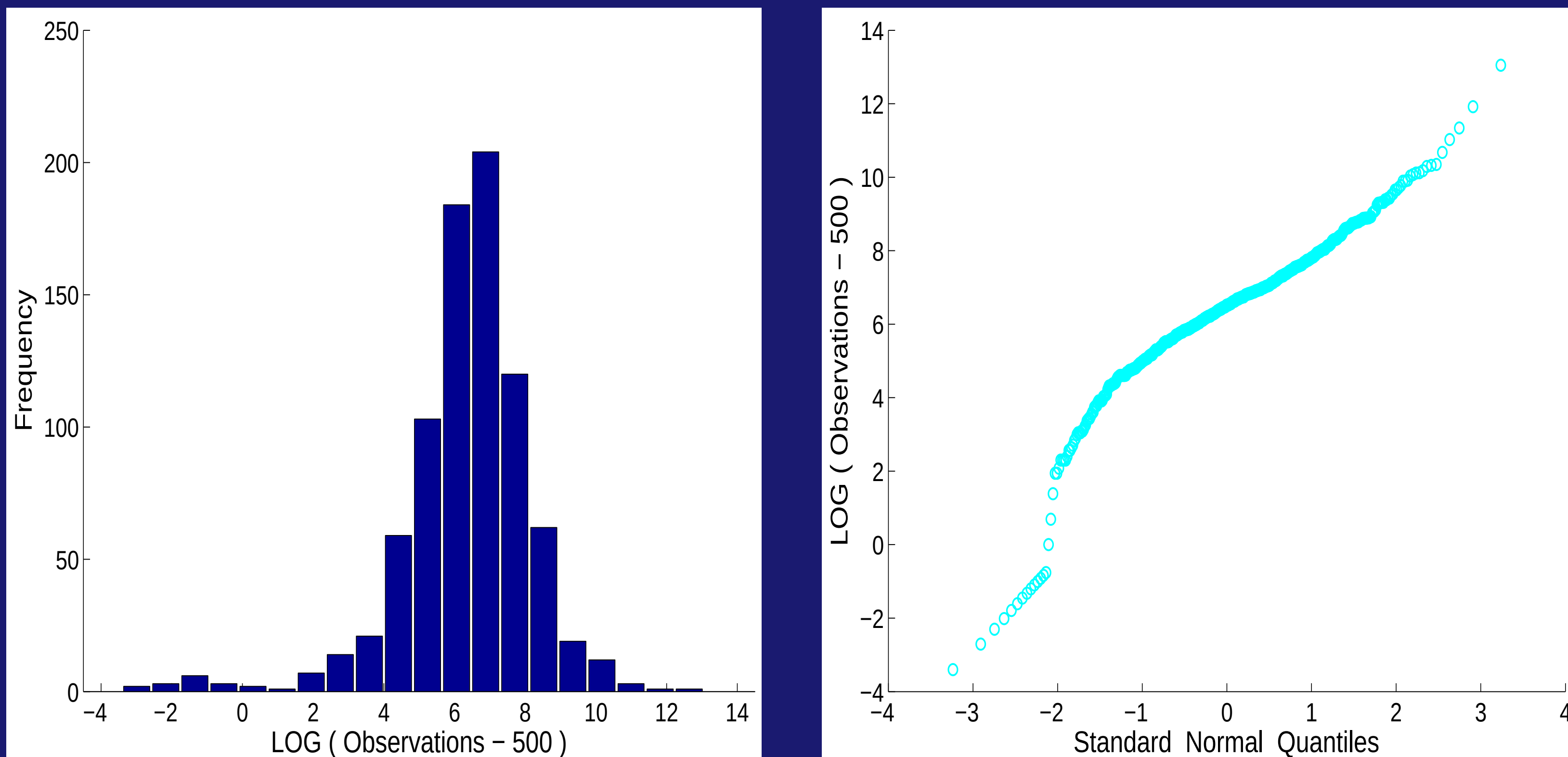


FIGURE 5: Preliminary diagnostics for the Norwegian data.

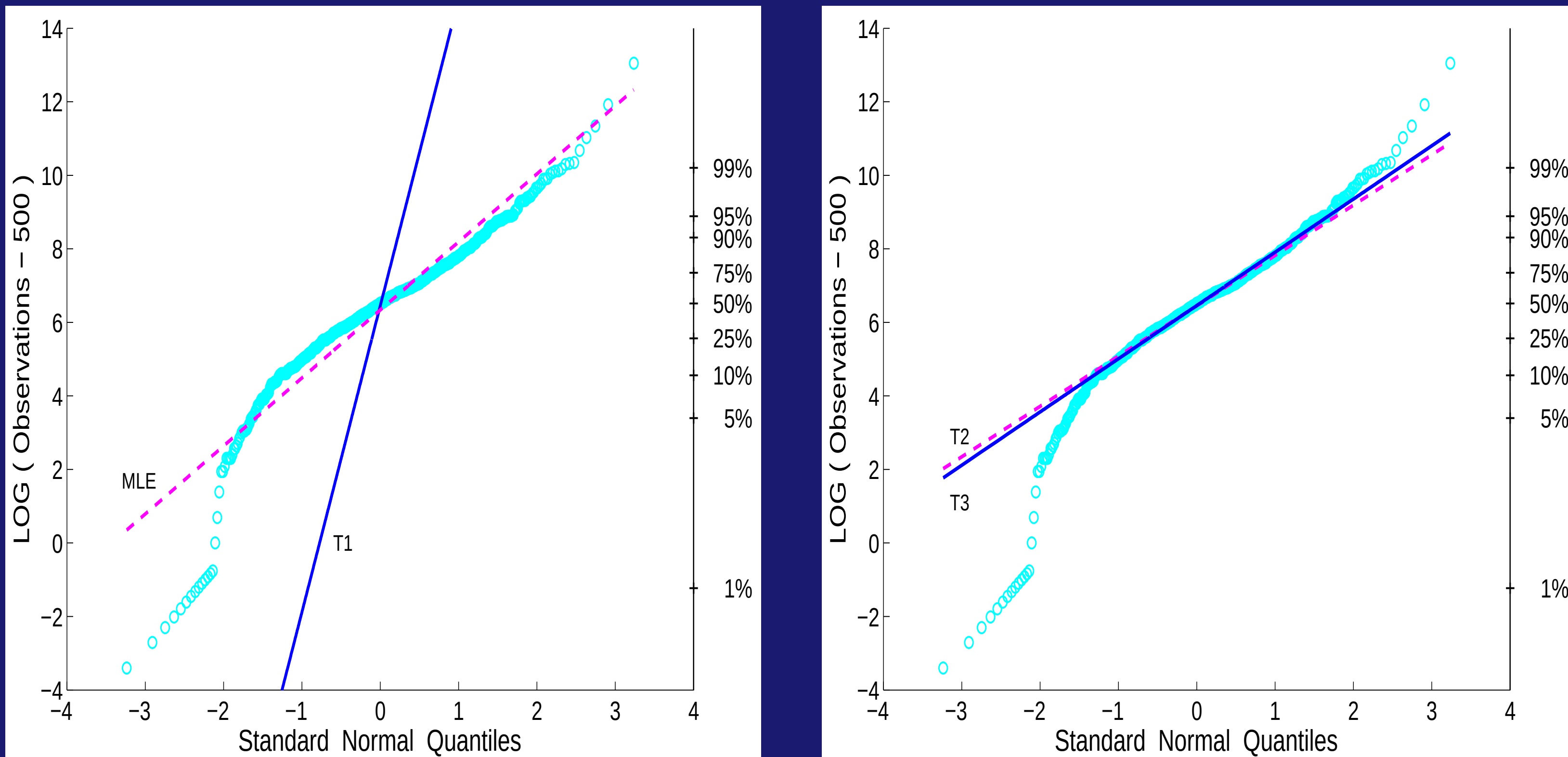
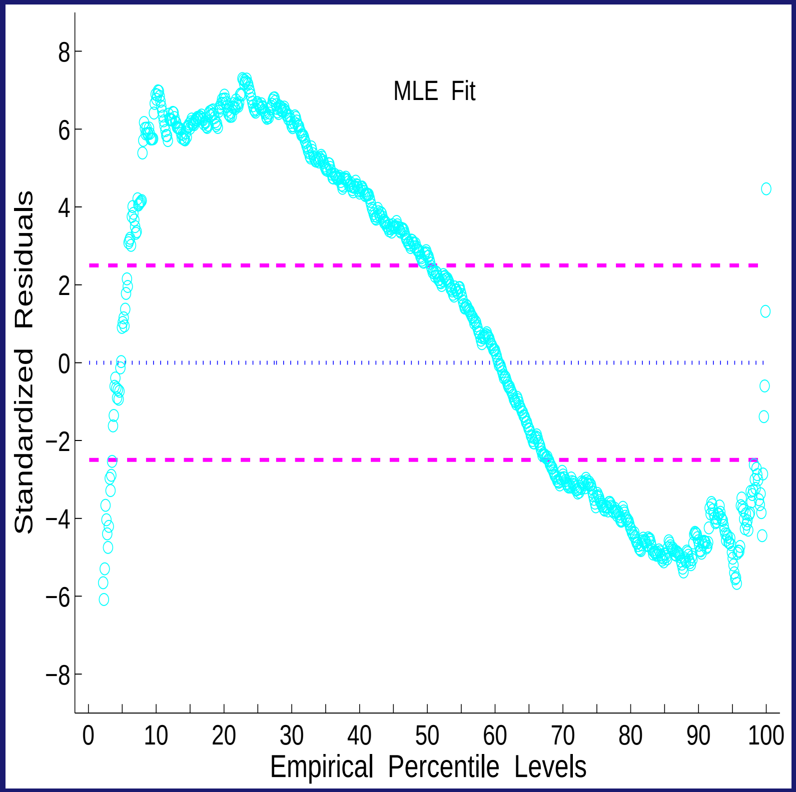


FIGURE 6: Lognormal QQP-plots and fits by MLE and MTM $(a,\, b)$.
T1: $(0.45,\, 0.45)$; T2: $(0.10,\, 0.10)$; T3: $(0.10,\, 0.01)$.



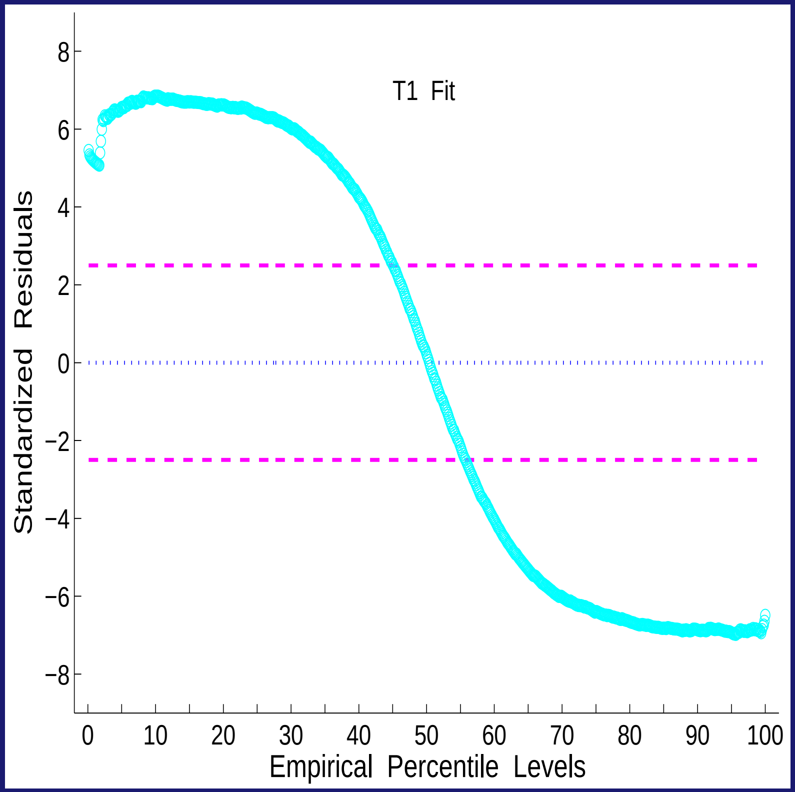


FIGURE 7: Lognormal PR-plots and fits by MLE and MTM (a, b).
T1: $(0.45, 0.45)$; T2: $(0.10, 0.10)$; T3: $(0.10, 0.01)$.

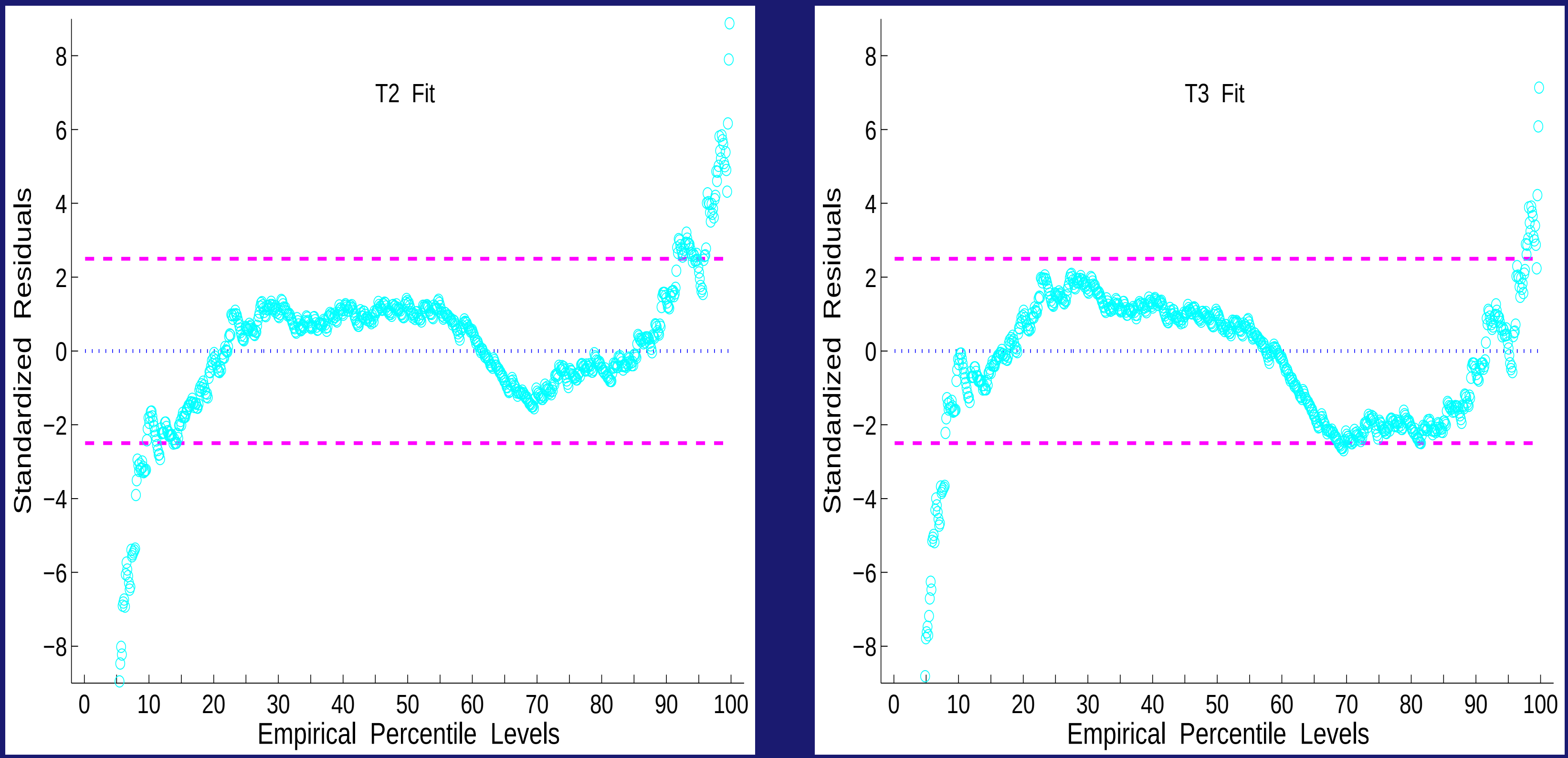


FIGURE 8: Lognormal PR-plots and fits by MLE and MTM $(a,\ b)$.
T1: $(0.45,\ 0.45)$; T2: $(0.10,\ 0.10)$; T3: $(0.10,\ 0.01)$.

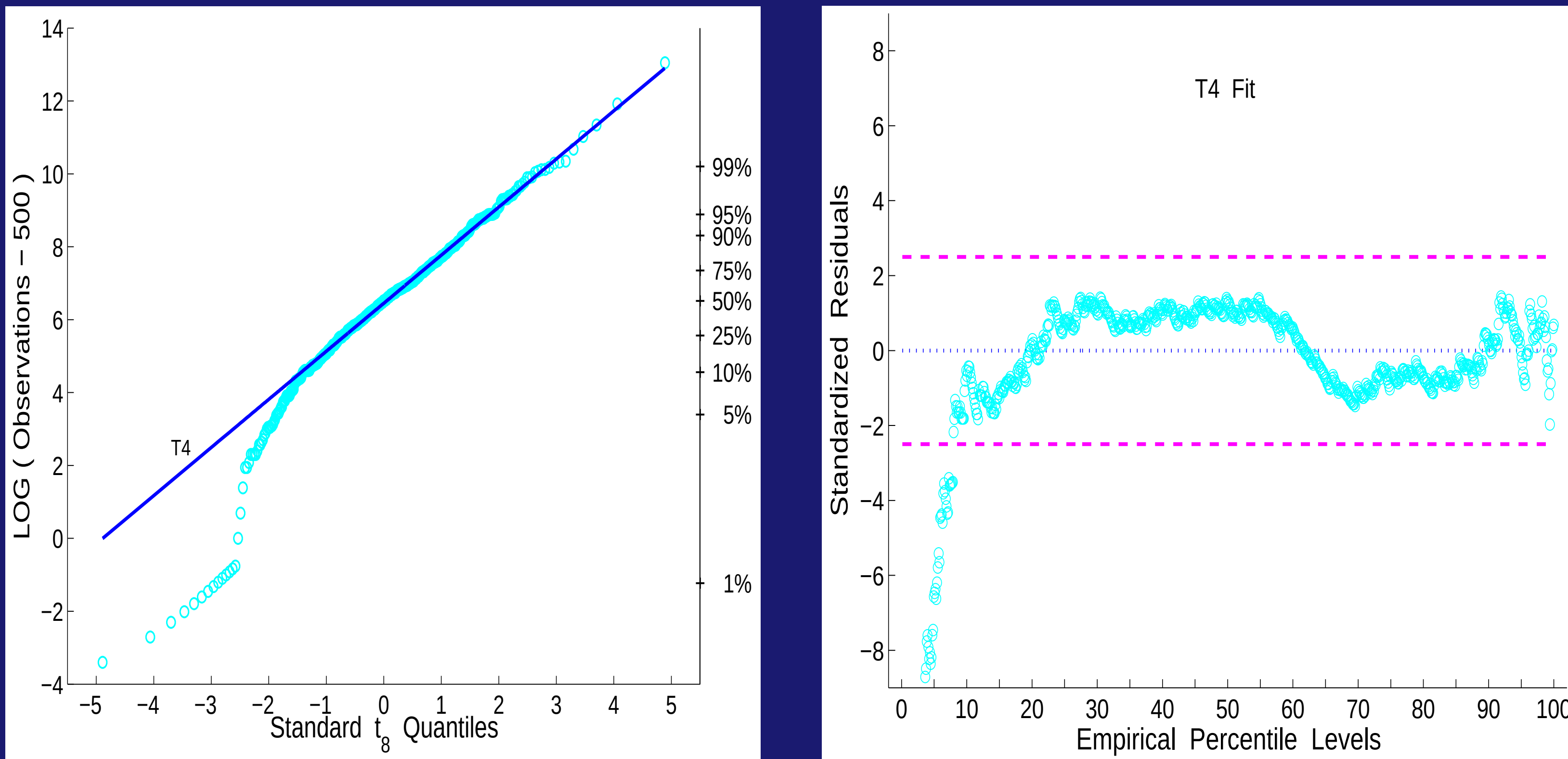


FIGURE 9: Log-t_8 QQP-plot and PR-plot. The log-t_8 model is fitted by the MTM method, with $a = 0.10$, $b = 0.01$ (T4).

TABLE 6: Point estimates and 95% confidence intervals of various value-at-risk, VaR(F, β), measures.

β	Estimation Methodology: EMPIRICAL	LOGNORMAL	LOG-t_8
0.25	2,058 (1,830; 2,268)	2,203 (1,960; 2,446)	2,112 (1,867; 2,357)
0.10	4,555 (3,758; 5,974)	4,607 (3,973; 5,242)	4,512 (3,821; 5,203)
0.05	7,731 (6,905; 11,339)	7,422 (6,244; 8,601)	7,850 (6,410; 9,290)
0.01	26,791 (20,800; 84,464)	18,856 (15,025; 22,686)	28,788 (21,360; 36,217)

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