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Pricing Disability Insurance Alongside Hedging Longevity Risk

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Foreword: Each calendar quarter, the Society of Actuaries (SOA) calls for essays that highlight practical applications of academic research published in the *North American Actuarial Journal*. This essay met the publication criteria of the judging committee. It outlines a practical application of "Hedging Longevity Risk: Does the Structure of the Financial Instrument Matter?"[1] by Richard D. MacMinn and Nan Zhu. The essay is solely an expression of the views, opinions, and ideas of the essay author for applying the original research to actuarial work in industry.

1. Introduction

Disability insurance (DI) provides benefits when one starts experiencing disabilities such as the following: respiratory diseases, neurological disorders, immune system disorders.[2] In general, DI products pay in the form of an *annuity* until the policyholder recovers from disability.[3] A crucial actuarial application within DI is *pricing*.

"Longevity risk" refers to the risk of an individual living longer than expected. This risk could be sub-divided into systematic[4] and non-systematic.[5] MacMinn & Zhu consider two common types of hedging instruments: cash flow hedging and value hedging. According to MacMinn & Zhu, a value hedge is preferred. However, it must be kept in mind that MacMinn & Zhu makes this conclusion focusing solely on how much shareholder wealth is preserved during the process of hedging.

This essay lends focus on understanding how the pricing application of DI can be better optimized with the introduction of hedging. In particular, this essay focuses on discussing the impact of pricing DI by using cash flow hedging instruments in contrast to using value hedging instruments.

[1] MacMinn, Richard D. and Nan Zhu. (2019, Nov. 22). "Hedging Longevity Risk: Does the Structure of the Financial Instrument Matter?" *North American Actuarial Journal*. https://www.tandfonline.com/doi/full/10.1080/10920277.2019.1650286.
[2] This is a non-exhaustive list. The entire list of disabilities would depend on the insurance provider, and the contract with the policyholder.
[3] This may change based upon the policies of the insurance provider, and the needs of the policyholder.
[4] Systematic risk refers to the uncertainty in the length of any person of a particular age cohort will live for.
[5] Non-systematic risk refers to the uncertainty in the length of any member in the insurance portfolio will live for.

1.1 CASH FLOW HEDGING VS VALUE-BASED HEDGING

A *cash flow hedge* is referred to as a series of payments to offset the liability of an insurer. On the other hand, a *value hedge* refers to a one-time payoff contingent on a publicly observable event. A key distinction here is how each hedge reflects future mortality.

Consider autism. A breakthrough in autism research could be considered a publicly observable event for a one-time payoff to be made from a value hedged instrument. However, this event does not take into consideration the fact that the level of future mortality sees significant turnarounds. This is where a cash-flow hedge comes into great effect. However, as discussed later, this seems sub-optimal from a shareholder value perspective.

2. Hedging with no Systematic Longevity Risk

In a world with no systematic risk, financial risk and non-systematic longevity risk takes charge. In such a scenario, a DI provider could use diversification to mitigate risk. Moreover, it has been depicted by that corporate management leans towards choosing to not hedge at all, in order to maximize shareholder wealth (MacMinn & Brockett, 2017). This is referred to as the *moral hazard problem.*[6] Further elaborating on this:

Risk mitigation is a concept that most corporate managers require focus towards. From the perspective of Enterprise Risk Management (ERM), it is critical that projects undertaken is in line with the company's risk appetite. However, when this element of risk is transferred to shareholders, corporate managers have incentive to increase the entity's risk exposure because it will not bear full cost of this extra risk. This is the moral hazard problem that inhibits corporate managers from hedging.

In general, solvency requirements are an integral part within the regulatory system. The insurer would be required to hold sufficient reserves such that

$$P(Bankruptcy) = \alpha$$

where "α" is set by the regulator.

Consider the following from an actuarial point of view. These are critical components when modeling premium.

$$\bar{a}^{00}, \bar{a}^{01}, \bar{a}^{11\,7}, \mu^{01}, \mu^{10};$$

0 – healthy state,

1 – state of disability

Uncertainty within the pool of internal policyholders insured for disability would certainly cause adjustments in the above components (and even more).

2.1 A CASH FLOW HEDGE WITH NO SYSTEMATIC RISK: IS IT OPTIMAL?

True, non-systematic risk could be eliminated using diversification. However, in my opinion, a cash flow hedge could be used to mitigate this non-systematic risk. A cash flow hedge instrument such as a mortality bond is able to de-risk the level of non-systematic risk of a DI provider. Consider Diagram 1.

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[6] This is when an entity has incentive to increase their risk exposure because it does not bear the full cost of that risk.

[7] We are interested in this value as longevity risk exposes lower mortality levels encouraging disabled policyholders to remain disabled for a longer period of time than expected.

Diagram 1

IMPLICATIONS OF USING A FULL CASH-FLOW HEDGE TO MITIGATE ONLY NON-SYSTEMATIC RISK

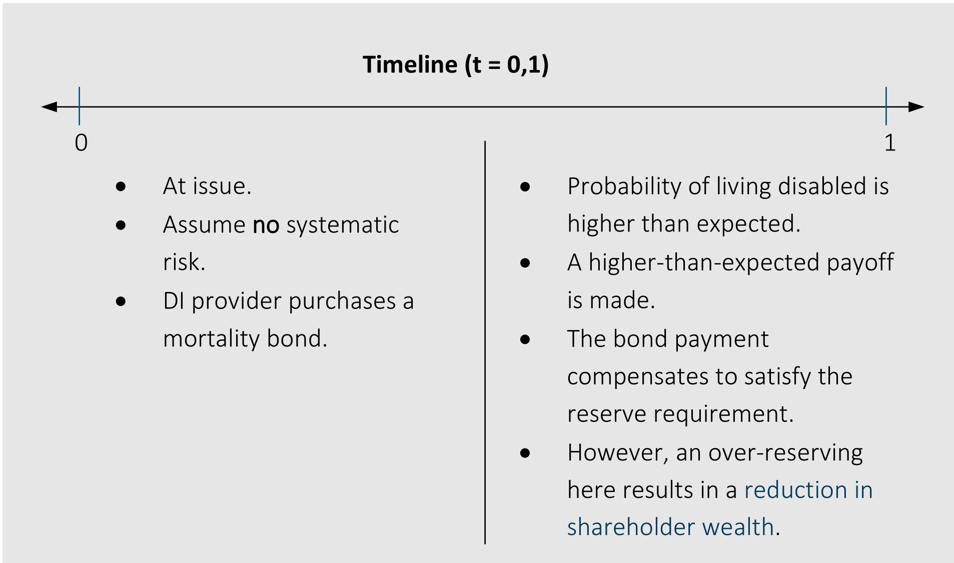

Diagram 1 demonstrates the implications of using a full cash flow hedge to mitigate only non-systematic risk. Recall that non-systematic risk refers to risk experienced *within the insurer's portfolio*. Further recall that a value hedge makes a payment contingent on a publicly observable event. As implied, an internal non-systematic risk event would not be due to a publicly observable event. This reveals that a cash flow hedge may seem optimal in contrast to a value hedge. As future mortality of the internal portfolio could further decrease, a series of payments does seem like the optimal choice to mitigate this risk. Nonetheless, a full cash flow hedge such as this can be deemed to reduce shareholder value and be sub-optimal from a shareholder perspective.

As presented in the diagram, a full cash flow hedge compensates for any non-systematic risk by making payments to ensure that the insurance company holds sufficient regulatory reserves. However, such payments tend to over-reserve, and thus results in the value of the company available to shareholders, to reduce (MacMinn & Zhu, 2019).

3. Hedging with Systematic Longevity Risk

A world with no systematic longevity risk is tough to imagine . Consider the following systematic risk events that could occur within the longevity market:

- A breakthrough within the automobile (air-travel) industry causing a great reduction in air pollution: this may cause *respiratory* morbidity rates to decrease significantly.
- Large amounts of funding allocated towards the research and development of solutions within the area of *neurological disorders*. Morbidity rates may need adjustments.
- Climate change initiatives to produce better quality air. This may impact *respiratory disease* morbidity rates in a positive manner.

As an actuary, implications from the future must be considered. Based on the risk tolerance level, together with the ERM framework of the insurer, adjustments must be made within pricing to reflect these possible risks[8].

DI products usually earn premiums when policyholders are healthy. This implies that the insurer must make sure premium rates are set, by accurately predicting $\bar{a}^{00}$. Moreover, uncertainty from systematic risk increases the probability of bankruptcy. To satisfy solvency requirements whilst preserving shareholder value, insurers are compelled to hedge.

MacMinn & Zhu consider two fundamental scenarios: with and without systematic risk. With a hedging instrument involved, assume an additional payoff of *C* to be paid out at time 1. This would adjust the expected payoff to the insurer as follows:

$$Payoff_{Insurer} = Payoff_{Insurer}^{No\ systematic\ risk} + (1-q) \times C = Payoff_{Insurer}^{With\ systematic\ risk} - q \times C$$

q – probability of a systematic risk event occurring

3.1 USING A VALUE HEDGING INSTRUMENT

Assume the occurrence of a systematic risk event – one that increases the probability of being disabled without moving to the state of death or back to being healthy – similar to one above. A value hedged instrument (a q-forward[9] for example) would result in the negative impact of your portfolio to be offset by the hedge. Assume the best-case *transition rates between states* use the market experience. In order to preserve shareholders value, a q-forward is purchased with forward rates lower than best-case rates. In the event of an increase in the transition rates from the state of *disable to disable*[10], the forward contract would result in positive gain to the insurer. This will offset the unexpected annuity payout to the policyholder during the period of disability. An event of opposing nature would act vice versa. Under the assumptions of a complete market, a value hedge would mitigate only the systematic risk component, but is able to satisfy the solvency requirement whilst preserving shareholder value.

3.2 USING A FULL CASH FLOW HEDGING INSTRUMENT

This involves a series of payments that depend on realized future mortality. This method usually makes payments on both systematic and non-systematic effects. Fundamentally this refers to swapping an uncertain payoff in exchange for a certain one (similar to an interest rate swap). An uncertain payout of A is exchanged for a certain payout of a by investing in a full cash flow hedge. The variability in cash outflows is mitigated 100% by using such an instrument (MacMinn & Zhu, 2019). Prior knowledge of this certain payment would assist the insurer to drop the probability of solvency close to zero.

$$Profit = Premium + Assets_{Reserves} - a$$

assume, a < Premium + Assets

Reducing the chance of the insurer running insolvent would mean that the put option on bankruptcy, from the perspective of the shareholders would be lost. Thus,

[8] How this is done will differ from company to company, and according to the demographics of their portfolio.
[9] A q-forward is a forward contract that involves the exchange of realized mortality for fixed mortality rates agreed to upon at inception. "q" refers to the probability of death in actuarial notation.
[10] Due to a lower mortality rate in the market, policyholders may remain disabled rather than transitioning to being healthy or being dead.

$$ShareholderValue^{FullCashFlowHedge} = ShareholderValue^{Unhedged} - PutOption$$

This reduction in value entails the notion that a full cash flow hedge does in fact clear out the insurer off any uncertainty in longevity, but at the extreme risk of facing unsatisfied shareholders.

3.3 USING A PARTIAL CASH FLOW HEDGING INSTRUMENT

In comparison, a partial cash flow hedge attempts to limit the transfer of value from shareholders to policyholders whilst satisfying regulatory requirements.

Instead of a pay-off of $(A - a)$, a payoff of $k \times (A - a)$ is made. This results in the time-1 payoff to the insurer to be:

$$Profit = Premium + Assets_{Reserves} + k \times [A - a] - A$$

This simplifies down to,

$$Profit = Premium + Assets_{Reserves} + A[k - 1] - k \times a$$

Modeling parameter k is what will assist the insurer in finding the right balance between the insurer's risk tolerance level, solvency requirements, and shareholder value.

Although the regulatory requirement could be satisfied, the level of shareholder value being preserved is still a point to take note of. As discussed earlier, a value hedge mitigates only systematic risk, whereas a cash flow hedge usually looks to balance both systematic and non-systematic risk.

A question arises as to whether k can be modeled in a way to preserve shareholder value, similar to in a value hedge. Consider the following results of a numerical experiment conducted by MacMinn & Zhu:[11]

α	$S^u = S^v$	k^*	S^c
1%	1075.77	22.68%	1075.49
5%	704.73	28.35%	702.55
10%	530.39	32.66%	524.63

α – the maximum probability of default as per regulation
S^u – shareholder value with no hedge in place
S^v – shareholder value with a value hedge in place
S^c – shareholder value with a partial cash flow hedge in place

As per the above, the difference in value between a value hedge and a cash flow hedge is narrowed down with a reduction in α.

[11] This experiment considered 462 combinations of financial & non-systematic risks. Directly taken in from the research paper (Table 2: page 11)

4. More on Relation to Pricing Disability Insurance (DI)

Pricing DI takes into consideration future disability benefits, transition rates, mortality rates, reserves, probabilities, expenses, other decrements, and company specific profit margins. Holding a high reserve implies a lower probability of bankruptcy. However, this would also imply a higher rate of premium.

Most insurance companies use stochastic methods to model premiums. In my opinion, and by considering the study presented by MacMinn & Zhu, a DI pricing actuary together with a specialized investment actuary could work in structuring a hedging instrument based on the insurer's level of risk. Thereafter, it could be connected to the pricing process accordingly.

Regardless of what application this study is being used for, it is critical to make note of the following point: the paper by MacMinn & Zhu views value from the perspective of the shareholder. There may be public sector companies, or even private firms that run on an entity focused business model. Moreover, the assumption of independence between financial and mortality risk is invalid within most practical spheres.

5. Conclusion

This essay discussed the implications of hedging longevity risk within the pricing application of DI. It demonstrated how a DI provider should pick between a cash flow hedged and a value hedged instrument to mitigate the risk of longevity.

Although this essay demonstrates that a value hedge is the better approach, the fact that this study focuses primarily on shareholder value should be considered when analyzing conclusions presented. Insurers that prefer alternate business models should use results discussed in this essay after making necessary adjustments.

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