ERM-INV Model Solutions
Spring 2022

1.   **Learning Objectives:**
   6.   The candidate will understand the concept of economic capital, risk measures in capital assessment and techniques to allocate the cost of risks within business units.

Learning Outcomes:
(6a)   Demonstrate a conceptual understanding of economic measures of value and capital requirements (e.g., EVA, embedded value, economic capital, regulatory measures, and accounting measures) and their uses in decision-making processes

(6b)   Apply risk measures and demonstrate how to use them in value and capital assessment

(6c)   Propose techniques of attributing the "cost" of risk/capital/hedge strategies to business units in order to gauge performance (e.g. returns on marginal capital)

(6d)   Demonstrate the ability to develop a capital model for a hypothetical organization

Sources:
ERM-106-12: Economic Capital – Practical Considerations – Milliman

ERM-119-14: Aggregation of Risks and Allocation of Capital (Sections 4-7 excluding 6.3)

Commentary on Question:
This question tests candidates' ability to understand capital models, aggregation of risks, attributing capital, and how they are used in capital assessments.

Solution:
(a)
   (i)   Identify and describe four of Acme's *key* stakeholders.

   (ii)   Outline the RC considerations of each key stakeholder, reflecting Acme's recent experience.

1. Continued

Commentary on Question:
This part asked candidates to identify and describe four key stakeholders. Most candidates did well on this part. Key stakeholders getting full credit included: Shareholders; Management; Policyholders; Regulators; Ratings Agencies; Employees; Board of Directors; Other Insurers; Lenders. Credit was given for the first four. Those closely related, i.e., Management and Employees was counted as only one key stakeholders. If more than four were listed, only the first four were graded. Candidates should tie to COVID experience.

(i)
Shareholders – Interested in returning a good return on invested capital. Not interested in extremes beyond ruin.
Management – Risk loadings in pricing, assessing performance, allocating capital.
Policyholders – Most interested in extreme events that threaten ability to pay claims.
Regulators – Interested in extreme events. Will want to see COVID impacts.

(ii)
Shareholders – would want small RC, but higher distributable earnings.
Management – Accurate RC as possible, to determine performance of each BU managers.
Policyholders – Would want a large RC, offering more protection.
Regulators – Would want a large RC, to protect against further COVID losses.

(b) Acme's management is considering a restructure, whereby some BUs would be divided and a new BU added. The proposed structure is described as follows:

- A is divided into two, unequal parts
- L is divided into two, identical parts
- P is unchanged
- N is the new business unit, which is considered risk-free.

The proposed new capital allocation is as follows:

BU	RC
A_1	95
A_2	4
L_1	30
L_2	20
P	15
N	5

1.   Continued

Assess whether the new allocation satisfies the requirements of a Coherent Capital Allocation methodology by comparing the proposed RC allocation to the existing allocation.

Commentary on Question:
Most candidates did well on this part. Candidates needed to state that this does not meet the requirements of a Coherent Capital Allocation method, and explain why using the three properties.

A Coherent Capital Allocation method must satisfy:

No Undercut – a sub-portfolio's allocation should be no more than its standalone capital requirement. This is satisfied since A1+A2<=100; L1+L2<=50

Symmetry – If the risk of two sub-portfolios is the same, the allocation should be the same. This is not satisfied since L1 and L2 are said to be identical, but have unequal capital.

Risk-free allocation – capital allocated to a risk-free BU should be zero. Not satisfied, as N is allocated 5.

This new allocation does NOT satisfy the requirements of a Coherent Capital Allocation method.

(c)
   (i)   Demonstrate that the amount by which total capital can be reduced with diversification is 24.58. Assume that the risks are normally distributed.

   (ii) Calculate the amount of RC for each BU using the Pro Rata (linear) approach. Show all work.

   (iii) Calculate the amount of RC for each BU using the Discrete Marginal Contribution approach. Show all work.

   (iv) Recommend either the Pro Rata or the Discrete Marginal Contribution method of allocation for Acme. Justify your answer.

Commentary on Question:
Candidates did very well on parts (i) and (ii). Some did not know the formula for (iii). Candidates received full credit on (iv) if they explained the difference between the two methods, made as recommendation, and explained why this was a better method for Acme in particular.

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1. **Continued**

Parts (i), (ii), and (iii) in Excel spreadsheet.

(iv)
I recommend the discrete marginal contribution approach for Acme. While the Pro Rata method is simpler, it is not as accurate at taking into account diversification benefits. Since management wants to reduce EC by taking into account correlations between business units, the Discrete Marginal Contributions approach would be more helpful in achieving that goal.

2. **Learning Objectives:**

3. The candidate will understand the concepts of risk modeling and be able to evaluate and understand the importance of risk models.

4. The candidate will understand how the risks faced by an entity can be quantified and the use of metrics to measure risk.

Learning Outcomes:

(3b) Demonstrate organization-wide risk aggregation techniques that illustrate the concept of risk diversification by incorporating the use of correlation.

(3d) Demonstrate the use of scenario analysis and stress testing in the measurement of current and emerging risks.

(3e) Demonstrate the importance of the tails of distributions, tail correlations, and low frequency / high severity events, and the use of extreme value theory to analyze these situations.

(4b) Analyze quantitative financial and non-financial data using appropriate statistical methods to assist in quantifying risk.

Sources:

ERM-142-20: Data Quality is the Biggest Challenge by Moody

ERM-120-14: IAA Note on Stress Testing and Scenario Analysis (pp. 1-6 and 14-17 and 19-25)

ERM-139-20: Quantitative Enterprise Risk Management by Mary Hardy, Chapter 7: Copulas

Commentary on Question:

This question assessed the ability of the candidate to apply rules and best practices to a real-world problem involving model design, data quality and stress testing. Candidates were tested on their ability to evaluate materiality of issues and use provided information as well as judgement to justify appropriate solutions. Most candidates scored well on parts (a) and (c), although most candidates did not receive full credit since they failed to consistently justify their responses. Most candidates scored poorly on part (b) as they appeared unfamiliar with Moody's Data Quality Practices.

2. **Continued**

Solution:

(a) The first objective of the external review is to assess the VaR model's design. Addison describes the methodology, data and assumptions as follows:

- The model uses parameters based on one year of historical market data as inputs to calculate VaR
- The calculation employs a Monte Carlo simulation and assumes that changes in risk factors follow a well-defined distribution, e.g., normal distribution or t-distribution
- To determine aggregated VaR, the model assumes correlations between the risk factors and a t-copula to incorporate increased correlation in the tail of the combined distribution
- The correlation assumptions are based on recent data and they have been backtested.

Critique the model design.

Commentary on Question:
Candidates generally scored well on this question although few received full credit. Several candidates failed to comment on all pieces of the model design in their critiques or they failed to explain/justify why a component was deemed appropriate or inappropriate.

- A common deficiency in calibration of distribution parameters is using a time-period of insufficient length to capture a representative range of conditions. One year of historical market data will therefore be insufficient and inappropriate for Addison's VaR calculations since most individual years will not include a wide range of conditions. Several years of data should be incorporated into the calibration to capture tail-type events if the focus of the analysis is on VaR.
- Monte Carlo design is appropriate for simulating investment returns since it is flexible and can capture complex non-linear relationships between variables. It may not always be the best methodology however, since even though it may show potentially disastrous results in the tail, it may not focus attention on the effect of the scenario to properly assess investment risk. Addison should ensure that her team can understand what is driving VaR results from the model and that the drivers align with the risk sources of interest for the Board. Also, Monte Carlo model designs can be resource intensive and require technical expertise. Addison should evaluate whether her team can appropriately maintain the model with their current resources.

2.   **Continued**

- A normal/t-distribution for changes in risk factors may not be appropriate for market risks that can be more right-tailed. Addison's team should consider other distributions and determine the best fit for the market behavior the team is trying to model.
- The use of a t-Copula is appropriate for addressing the lack of sufficiently heavy tails in the probability distributions used. However, although the associated correlation assumptions may introduce heavier tails, there is always the possibility that they don't capture sufficient tail risk. Also, using a copula will require even more expertise since calibration can be complex and explaining the use of a copula may be difficult for the Board to understand. Addison should ensure her team has sufficient understanding and resources to calibrate the copula and provide necessary explanations.
- Basing correlation on recent data only, could be inappropriate, depending on whether or not recent relationships are reflective of the investment risk Addison's team is trying to model.
- Back-testing however, is one appropriate tool for validating the correlation assumptions being used. After backtesting is conducted, the team will have to use their judgement to determine if recent data should continue to be used or if more historical data should be added. As Addison's team reports model results, caution should always be suggested as history is not always a predictor or future results.

(b)   The company has an internal requirement to follow Moody's Data Quality practices.

   (i)   Describe the most critical step in the Moody's recommended seven-step quality process that is not being followed. Justify your response.

   (ii)   Describe the two most critical data quality rules applicable to this data. Justify your response.

Commentary on Question:
Candidates generally scored poorly on this part of the question because they did not appear to recall Moody's seven-step quality process or Moody's data quality rules. Several candidates incorrectly referenced EIOPA Data Quality Requirements from the same reading. Also, both subparts asked the candidate to describe the most critical steps. Partial credit was awarded for listing other steps/rules that were critical (but not the most critical), but only if sufficient justification was provided. In order to get full points in both subparts, justification of "most critical" needed to tie back to the issues listed and describe how the recommended step / data quality rule addressed the issue.

2. **Continued**

    (i)   The most critical step they are failing to follow in Moody's seven-step data quality process is step 4, Data Standardization. Data standardization involves execution of a series of data quality checks and rules against the monthly data being appended. Best practice is to execute the data quality rules within the repository. In this case, Addison's team needs to check the format and quality of the data every time an update is made and implement a rule to ensure that it is consistent with previous data the team has stored in their repository. Currently, the monthly change in the formats of the reported data is likely leading to the large VaR fluctuations observed from month to month. Data standardization will address the issue.

    (ii)   The two most critical data quality rules that apply to Addison's monthly data are Range Constraints and Set-Membership Constraints.

- Range constraints will set minimum or maximum allowable values for the data based on typical ranges for the data type. In this case, negative expected returns are showing up in the data. The range should be restricted to exclude negative values to address this problem. Addison's team should also make note of any negative data points and investigate the issue.
- Set-Membership constraints will ensure values for a column come from a set of discrete values or codes. In this case, the rule is already being used for the risk type field, but the set of discrete values needs to be updated to include the 4th risk factor. Once the set-membership constraint is updated, the issue will be addressed.

(c)   Addison has decided to begin with a sensitivity analysis where there is a 5% decline in the equity markets. She would use the resulting impact to the investment portfolio to enhance the reporting to the Board.

    (i)   Critique Addison's decision.

You tell Addison that other scenario types should also be considered. Addison returns with the following scenarios:

- Single Factor Scenario – an unexpected spike in claims that requires the liquidation of 10% of the investment portfolio at current market prices
- Multi-Factor Stress Scenario – a 1-year recession that depresses the market value of equities in the portfolio, results in defaults in their bond portfolio and drives inflation resulting in increased claim costs and a slight dip in business volume late in the year

2. **Continued**

- Multi-Factor Multi-Period Stress Scenario – a severe, 2-year market downturn that increases the severity of the Multi-Factor Stress Scenario. Additionally, the scenario reflects a significant reduction to investment income and business volumes in year 2.

(ii)   Evaluate each scenario type given the Board's needs and Addison's available resources.

(iii)   Recommend the most appropriate scenario type based on your analysis in part (ii). Justify your response.

Commentary on Question:
Candidates generally scored well on part (c). Most candidates received partial credit for subpart (i), but few received full credit since their critique of Addison's decision didn't tie back to all of the Board's requests. Several candidates received close to full credit for subpart (ii). Responses commonly fell short in consistently addressing all three components of the Board's requests for each scenario type. Candidates received full credit for subpart (iii) if their response included sufficient justification and it was consistent with subpart (ii). Responses for subpart (iii) were not contingent on answering subpart (ii) and candidates could receive full credit for subpart (iii) even if their response for subpart (ii) was incorrect or incomplete.

(i)   The Board has expressed a desire to better understand specific conditions that could result in tail losses and whether there would be any associated long-term impacts to their business. Addison is proposing the use of a single factor sensitivity test. Typically, Single Factor Sensitivity tests are low in complexity which makes them efficient for both time and resources and this is a positive for Addison since she appears to have limited resources. However:
- Single factor sensitivity tests are typically low in explanatory power
- A 5% drop in equities is not a severe stress scenario, as evidenced by recent events
- Equities are not the main asset of the company

Based on the above, the sensitivity test Addison has proposed will not enhance the reporting to the Board. To address the Board's needs, she should also incorporate a drop in the investment grade bond values as well as increased defaults to show something that can threaten the company's financial stability. She should also consider a sustained shock to interest rates that could affect the company's business plan, to reflect potential long-term impacts.

2. Continued

(ii) Each scenario type should be evaluated based on its ability to:
- Assess tail risk
- Incorporate stresses on multiple variables to address the investment portfolio and business impacts
- Evaluate these impacts sustained over an extended time-period since business impacts may take longer to materialize.

Once Addison has assessed the ability of each scenario type to address the Board's needs, she should also reflect on her current resources and the level of sophistication necessary to evaluate the scenarios.

Single Factor Scenario – Generally, these scenarios are used for medium stresses and are low in explanatory power. Although this scenario will help the company evaluate liquidity risk, it does not represent a tail scenario for investment risk. It also fails to capture any long-term business impacts. It will not address the Board's needs. It is a straightforward scenario however, and Addison's team should have sufficient resources and the sophistication necessary to implement the scenario.

Multi Factor Stress Scenario – Generally, these scenarios are high in explanatory power and consider multiple risk factors. A recession that incorporates equity losses and bond defaults appropriately represents a tail scenario. The scenario also incorporates business impacts through increased claim costs and reduced business volume, but they are only short term as it specifies 1 year. This scenario does a better job of addressing the Board's needs, but doesn't satisfy all of the requirements. Relative to the single factor scenario, the multi factor scenario will require more resources and sophistication to evaluate. This is likely manageable for Addison's team, but may begin to be a strain on their resources.

Multi Factor Multi Period Stress Scenario – These scenarios are typically highest in explanatory power and consider multiple risk factors over extended time periods. This scenario clearly addresses all of the Board's concerns. Relative to the other scenarios considered, the multi factor multi stress scenario will likely require significant resources and sophistication to evaluate. This will be difficult for Addison's team to manage.

2. **Continued**

    (iii)   Based on the analysis in part (ii), I would recommend the Multi Factor Stress Scenario. It goes further than the Single Factor Scenario in addressing the Board's needs and understanding. It will also require considerably less resources than the Multi Factor Multi Stress Scenario. Although my recommendation doesn't address all of the Board's concerns it's appropriately balanced with Addison's current resources and level of sophistication, making it the best solution in the short term.

3. **Learning Objectives:**
1. The candidate will understand the ERM framework and process and be able to apply them to organizations.

2. The candidate will understand the types of risks faced by an entity and be able to identify and analyze these risks.

3. The candidate will understand the concepts of risk modeling and be able to evaluate and understand the importance of risk models.

4. The candidate will understand how the risks faced by an entity can be quantified and the use of metrics to measure risk.

6. The candidate will understand the concept of economic capital, risk measures in capital assessment and techniques to allocate the cost of risks within business units.

Learning Outcomes:
(1b) Demonstrate an understanding of the perspectives of regulators, rating agencies, stock analysts, auditors and company stakeholders and how they evaluate the risks and the risk management of an organization.

(2c) Identify and analyze specific risks faced by an organization, including but not limited to: financial, environmental, operational, legal, reputational and strategic risks.

(3d) Demonstrate the use of scenario analysis and stress testing in the measurement of current and emerging risks.

(3e) Demonstrate the importance of the tails of distributions, tail correlations, and low frequency / high severity events, and the use of extreme value theory to analyze these situations.

(4c) Analyze risks that are not easily quantifiable, such as liquidity, operational, and environmental risks.

(6a) Demonstrate a conceptual understanding of economic measures of value and capital requirements (e.g., EVA, embedded value, economic capital, regulatory measures, and accounting measures) and their uses in decision-making processes

Sources:
ERM-106-12: Economic Capital-Practical Considerations-Milliman

ERM-120-14: IAA Note on Stress Testing and Scenario Analysis

3. **Continued**

ERM-137-20: ORSA and the Regulator by AAA

ERM-138-20: Quantitative Enterprise Risk Management by Mary Hardy, Chapter 6: Extreme Value Theories

SOA Monograph: A New Approach to Managing Operational Risk, Ch. 8

Commentary on Question:
This question tests a candidate's understanding of the ORSA report, specifically as it relates to Lyon from the Case Study. The Candidate is expected to know details on Operational Risk, and the candidate should be able to do an Extreme Value Theory analysis. The question used Describe for Comprehension assessment and used Evaluate, Explain or Justify for Analysis or Knowledge Utilization assessments. In a Fellowship level exam, the Candidate is expected to spend more effort in their explanations, drawing on the curriculum while using specific references to Lyon from the Case Study.

Solution:
(a) The following feedback was provided by the regulator:

"A key weakness of Lyon's approach to the group capital assessment of the enterprise in the ORSA is the lack of group-level analysis."

Explain the issue raised by the regulator using two relevant examples from the Case Study.

Commentary on Question:
Candidates did reasonably well on this part, explicitly and appropriately referencing the Case Study with examples and sufficiently providing possible issues.

From page 48 of the Case Study:
"Excerpt from the Capital Assessment section of Lyon's ORSA Report:
Lyon determines its capital requirements based on the economic capital process that is already in place within its subsidiary companies."
The concern here is the lack of consistency across the company, on how risks are calculated and aggregated. This demonstrates an ERM program that is not mature.

3. Continued

From page 49 of the Case Study:
"The acceptance of risk is the primary responsibility of the subsidiary. Risk is first identified, measured and managed at the subsidiary entity level. Diversification across risk types is calculated at the subsidiary level. Risk aggregation to the corporate level is the sum of all subsidiary-level risks by risk category.

> Lyon is ignoring intra-group creation of capital and contagion risks, such as reputational or strategic risk, or missing risk diversification benefits.

(b) Another key issue raised was that Lyon's Corporate capital quantification for credit risk is not modeled, even though credit risk is a key risk for the enterprise.

A consultant was hired to develop loss scenarios on the Corporate asset portfolio. He produced the following results.

Percentile	Loss (in 000s) at end of year
75%	$5,450
90%	$9,681
95%	$14,368
98%	$17,725

Evaluate the reasonableness of Lyon's Corporate Economic Capital value for Credit Risk given these scenario results.

Commentary on Question:
Candidates were expected to evaluate critically Lyon's Corporate Credit Risk value from the Case Study or the scenario results in relation to that specific value. In some cases, candidates either referenced the wrong value and/or the wrong units or failed to make a substantial evaluation.

From the financials in section 2.11, Lyon's Corporate Credit Risk EC is $18.58 million, developed by a simple factor-based approach. The scenarios provided show a potentially fat-tailed distribution with significant recession-level results (90-95%ile) and 98%ile that's nearly $18 million already.

Given credit risk is a concern of the group as well as the subsidiaries, Lyon should develop a consistent modeling approach across the enterprise. This would replace the factor-based approach for the Corporate Credit EC and would provide a better view of how much of the $178 million available EC is excess capital.

3.   **Continued**

(c)   Senior management has requested that the Corporate Risk Committee develop improvements to the ORSA for next year's submission.

   (i)   Explain how stress and scenario testing can improve Lyon's ERM framework and ORSA reporting.

   (ii)   Describe two items that regulators will consider when evaluating stress testing in the ORSA report.

Commentary on Question:
Candidates generally did well on this part, though answers should have been more specific to Lyon in part (i).

   (i)   Scenario and Stress Testing evaluates the strength of the business by subjecting it to varying combinations of economic, liquidity, operational, reputational conditions with differing degrees of severity. Lyon is in the process of developing an aggregate risk appetite statement which could be informed by enterprise-wide stress and scenario testing.

   Linking risk drivers to actual historical events, for example, a recession or pandemic, can be helpful in improving senior management and board understanding of risks and then they can develop mitigating strategies.

   (ii)   Regulators want to understand the severity of the stresses, including how the scenarios and stresses were calibrated, including any model and assumption validations. They would also like to see reverse stress tests or "break the bank" scenarios.

   Regulators want to know the completeness of stress testing, such that it includes risks material to the enterprise and considers combined risks and interrelationships among risks.

3. Continued

(d) Senior management has requested several enhancements to the group capital assessment and scenario/stress testing. The first recommendation is to develop an Operational Risk provision for the enterprise.

    (i) Describe two quantitative approaches to modeling Operational Risk that would be appropriate for Lyon.

    (ii) Explain the considerations of the following as they relate to data used in modeling Operational Risk:

- Internal vs External data
- Hard vs Soft data.

For calculating Operational Risk capital, a colleague suggests using a Generalized Pareto Distribution with a threshold at $1 million, the calibrated shape parameter at 0.80 and the calibrated scale parameter at 0.5.

$$Q_a = d + \frac{b}{k}\left(\left(\frac{S_X(d)}{1-a}\right)^k - 1\right)$$

There are a total of 500 loss values collected across the businesses. You are provided the following table showing the top 30 largest losses, in millions of dollars.

12.33	8.71	6.74	4.41	4.20
3.31	2.97	2.65	2.58	2.40
2.29	2.21	2.12	1.89	1.76
1.35	1.34	1.28	1.27	1.25
1.15	1.13	1.10	1.07	1.05
0.99	0.97	0.96	0.92	0.91

    (iii) Calculate the 99th percentile of operational losses using the GPD. Show all work.

    (iv) Evaluate the result from (iii) and discuss how it relates to the economic capital held by Lyon. Justify your answer using information from the Case Study.

3. **Continued**

> **Commentary on Question**:
> *Candidates did generally well with the shorter describe and calculate parts, but explanations and evaluations were limited. Some candidates choose to calculate the 99%ile using a different technique than the GPD requested or miscalculated the value, but if the explanations in part d(iv) were reasonable relative to the value in d(iii), credit was still given for that part.*

(i)
- Simple add-on model where the factor can be derived from costs at a certain confidence level with a certain degree of correlation would be easy to implement at Lyon.
- Monte Carlo Frequency/Severity simulation using stochastic scenarios developed from historical data and expert judgement would be better but take more time to develop and run.

(ii)
- Internal vs External data:
  Internal data can be collected directly by the company and reflect pertinent losses. However, there might not be adequate internal data to model and analyze operational risks in the tail. Therefore, external data should be used to supplement the data. However, external data needs to be reasonably unbiased, reflect similar operations of Lyon and is scaled to a level comparable to Lyon.
- Hard vs. Soft data:
  Hard data is collected intentionally on a prospective basis for short period of time and produces far more data points than soft data. For example, hard data would represent the day-to-day operational failures that tend to be high frequency but low severity. However, hard data does not capture tail events and therefore is not appropriate to use alone. Soft data collects fewer events looking back over time, but does capture extreme events, and it is the most appropriate to use when modeling/incorporating tail events. When modeling operational risks, it will be critical to include the low severity, high frequency events captured in soft data.

(iii)
> $Sx(d) = 25/500 = 0.05; \geq 95\%$, so EVT appropriate
> d=1.0, and k = 0.80 and b = 0.5 (given)
> 1 + (.5/.8)((0.05/.01)^.8-1) = $2.64 Million

3. **Continued**

(iv)

Based on the result, we expect that losses from a 1-in-100 Operational event to be greater than $2.64M. The result is between 10%-20% of Lyon's $18M Corporate EC, but that's only for Credit Risk and is too small if intended to be the Ops EC for the whole enterprise . This internal data from the businesses could be supplemented with external or soft data to reflect larger potential losses and then re-parameterize the GPD.

4. **Learning Objectives:**

1. The candidate will understand the ERM framework and process and be able to apply them to organizations.

2. The candidate will understand the types of risks faced by an entity and be able to identify and analyze these risks.

4. The candidate will understand how the risks faced by an entity can be quantified and the use of metrics to measure risk.

5. The candidate will understand the approaches for managing risks and how an entity makes decisions about appropriate techniques.

6. The candidate will understand the concept of economic capital, risk measures in capital assessment and techniques to allocate the cost of risks within business units.

Learning Outcomes:

(1c) Demonstrate how to articulate an organization's risk appetite, desired risk profile, quantified risk tolerances, risk philosophy and risk objectives.

(2c) Identify and analyze specific risks faced by an organization, including but not limited to: financial, environmental, operational, legal, reputational and strategic risks.

(4a) Determine risk exposures using common risk measures (e.g., VaR and TVaR) and compare the properties and limitations of such measures.

(4c) Analyze risks that are not easily quantifiable, such as liquidity, operational, and environmental risks.

(5e) Determine an appropriate choice of mitigation strategy for a given situation, which balances benefits with inherent costs (including exposure to moral hazard, credit, basis and other risks).

(5i) Choose appropriate techniques to measure, model and manage various financial and non-financial risks faced by an organization.

(6a) Demonstrate a conceptual understanding of economic measures of value and capital requirements (e.g., EVA, embedded value, economic capital, regulatory measures, and accounting measures) and their uses in decision-making processes

4. **Continued**

Commentary on Question:
This question was trying to test candidates knowledge of liquidity risk. The question included two calculations, both involving asset liquidity measures and the resulting impact on the financials of the company. Related subparts required an understanding of the results of the calculations as they related to liquidity considerations.

Parts of the question not related to the calculations required knowledge of liquidity risk frameworks and potential areas of risk. Candidates who scored higher on these parts tended to provide answers in the context of the liquidity risks specifically facing a health insurance company.

Solution:
(a)   In considering which asset to sell, the CIO asks you if any of the four assets have breached the individual asset liquidity limit. Assume a normal distribution and critical value of 1.645.

     (i)   Determine which, if any, of the individual assets have breached the asset liquidity limit. Show all work.

     (ii)   Recommend which asset to sell based on the results in (i). Justify your response.

     Commentary on Question:
     Subpart (i)
     Many candidates received full credit on these calculations and the interpretation of the results. Those that didn't tended to have errors in the LVAR formulas, or did not demonstrate an understanding of the breach limits (i.e. ratios above 35%).

     Subpart (ii)
     Candidates typically received either full credit or no credit here, which correlated highly with correct interpretation of the individual asset liquidity limits and whether or not assets were in breach of those limits.

4. Continued

    (i)   Refer to Excel for calculation solutions.

    (ii)   Both the BB Corporate bond and the B corporate bond have breached the individual asset equity limit. The BB Corporate bond should be sold, as its ratio (37.6%) exceeds the 35% limit by more than the B Corporate bond's ratio (35.5%).

(b)   Question statement.
ABC uses the NAIC Health RBC formula when calculating the RBC ratio. The following table shows book values and average post-tax RBC factors by risk category used by ABC to calculate its RBC ratio.

RBC Category	Book Value	Average post-tax RBC Factor
H0	1,299	0.030
H1	23,612	0.022
H2	32,000	0.044
H3	12,478	0.032
H4	2,220	0.050

Total Adjusted Capital (TAC) = 5,527

    (i)   Describe what RBC is intended to measure.

    (ii)   Assume that cash held on the balance sheet has a 0% RBC factor.

              Analyze the impact on the company's RBC ratio of selling each individual asset from part (a). Show all work.

    (iii)   Discuss whether your analysis performed in part (ii) has changed your prior recommendation to the CIO on which asset to sell. Justify your response.

Commentary on Question:
Subpart (i)
Many candidates received either no credit or nearly full credit. Those who did not receive credit failed to mention that RBC is a regulatory tool that can lead to regulatory action, and not a standalone tool to measure the competitiveness or financial solvency of an insurance company.

4. **Continued**

(i)   RBC is a standard regulatory formula that represents the minimum capital requirement for insurers, and is used to determined when a regulator must take action against an insurer, up to and including taking control. RBC is not the amount of capital that an insurer would want to hold to meet growth or competitive objectives.

(ii)   Refer to Excel for calculation solutions.

(iii)   Based on the impact to the RBC ratios, the B Corporate bond should be sold. While the LVAR/VAR ratio for the BB Corporate bond is higher, selling the BB bond decreases the RBC ratio from 345.0% to 344.7%. Conversely, selling the B bond increases the RBC ratio to 345.5%.

(c)   The CIO wants to review the company's liquidity contingency plan.

(i)   Outline the main components of a well-designed liquidity contingency plan as part of a broader liquidity risk framework.

(ii)   Explain how a catastrophic event could impact the liquidity needs of ABC.

(iii)   Identify three factors, other than a catastrophic event, that could impact the level of ABC's available liquidity.

Commentary on Question:

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4. **Continued**

Subpart (ii)
The large majority of candidates received at least some credit on this part. Almost all candidates received credit for recognizing that a catastrophic event would lead to an increase in claims, but many failed to identify any potential impacts beyond that. This question was specifically testing candidates understanding of liquidity needs, and not liquidity resources.

Subpart (iii)
Many candidates received at least half of the potential credit on this question, though a large number received none. Those that received less credit failed to identify factors that could affect the available liquidity of a health insurance company, and instead provided responses that were not applicable to ABC.

(i)   The main components of a liquidity contingency plan include:
- procedures to detect a potential liquidity stress event using early warning indicators
- a pre-defined list of contingency resources in the case of a stress event
- procedures and governance to activate the contingency plan
- procedures and governance to decide when to implement the appropriate response to a liquidity stress event, and monitor the implementation
- the process for internal and external communication
- the process to periodically test and update the plan

(ii)   Catastrophic events trigger an increase of insurance payment obligations, leading to large unexpected liquidity needs or payment requirements within a short time. Catastrophic events may also trigger drastic changes in consumer behavior such as mass lapses, or could result in the downgrade of assets, making them ineligible for collateral posting.

(iii)   Other factors that could impact the level of ABC's available liquidity include:
- A reduction in the market value of assets
- Limited access to external and internal funding
- Reduced funding from business operations (e.g. lower than expected inflow of premium income from new business)

5.

Learning Objectives:

3.   The candidate will understand the concepts of risk modeling and be able to evaluate and understand the importance of risk models.

5.   The candidate will understand the approaches for managing risks and how an entity makes decisions about appropriate techniques.

Learning Outcomes:

(3d)   Demonstrate the use of scenario analysis and stress testing in the measurement of current and emerging risks.

(5e)   Determine an appropriate choice of mitigation strategy for a given situation, which balances benefits with inherent costs (including exposure to moral hazard, credit, basis and other risks).

(5g)   Analyze how ALM and other risk management principles can be used to establish investment policy and strategy, including asset allocation.

(5i)   Choose appropriate techniques to measure, model and manage various financial and non-financial risks faced by an organization.

Sources:

ERM-144-20: IAA Risk Book - Chapter 13: Asset Liability Management Techniques and Practices for Insurance Companies (LO 5)

ERM-112-12: Revisiting the Role of Insurance Company ALM within a Risk Management Framework (LO 5)

Financial Enterprise Risk Management, Sweeting, 2017 Ch. 16 Responses to Risk (LO 5)

Commentary on Question:

Commentary listed underneath question component.

Solution:

(a)   During your initial meeting with the company, Magenta Rock's CFO, Juan, states: "I believe we spend too much time worrying about the duration gap between our assets and liabilities. Given that Magenta Rock reports its financials on a book value basis, the primary focus of the ALCO should be to minimize absolute risk from asset volatility and minimize accounting volatility."

     Evaluate the assertions made by Juan.

5. Continued

While book value and accounting volatility are important considerations, they are more related to a short-term view for regulatory and reporting purposes.
Economic value should also be considered as it takes a long-term view on growth and profitability for the company.
The fact that the effective duration of liabilities is much longer than that of the assets may result in significant problems with funding risk especially in times of economic downturn with adverse interest rate changes. With liabilities having higher durations, the company will need to continually search for and roll new assets into its portfolio. When market stressed environments, it may be difficult or impossible to both find assets with sufficient duration as well as having to sell existing assets at depressed prices in order to meet cash outflow obligations.

(b) At a follow-up meeting, the ALCO expresses concerns regarding the current low interest rate environment and the volatility of the economic surplus. The ALCO suggests immunizing Magenta Rock's portfolio.

 (i) Describe two ways in which Magenta Rock would still be exposed to interest rate risk even if the portfolio is immunized using effective duration.

 (ii) Recommend a strategic asset allocation framework that would help increase Magenta Rock's portfolio yield while still addressing its concerns of surplus volatility. Justify your response.

b(i)
- Immunizing may work effectively only for small change of interest rate. If the volatility of interest rate is large, it may still face the risk.
- Sometimes it may be hard to find assets with long enough duration to fulfill the immunizing in the actual market. So, it may still face the interest rate risk in future.

5. **Continued**

b(ii)
Magenta Rock should constrain surplus volatility but relax the duration matching requirement. The surplus volatility will help to ensure assets are consistently sufficiently above liabilities. Relaxing the duration constraint will allow Magenta rock to seek higher yields through potentially riskier equity or alternative investment vehicles.

Note that Magenta could also look for assets with higher times to maturity which, in an upwards yield sloping environment would yield higher returns and would actually help with duration matching given the liability portfolio's long duration.

(c)   To further test the impact of the current interest rate environment, an ALCO member suggests that the following scenario test should be conducted:

A 50 basis point parallel drop in the risk-free rates for one year followed by an additional 10 basis point drop for each of the next four years.

   (i)   Assess how Magenta Rock's ALM profile would be impacted under the stress test circumstances.

   (ii)   Recommend one additional method Magenta Rock could use to measure interest rate risk to complement the scenario testing. Justify your recommendation.

Commentary on Question:
Candidates scored reasonably well on both parts of 5(c)

c(i)
*   On the asset side, the asset value will increase due to lowered yield curve, but it will suffer from reinvestment risks from any bonds maturing in the near future
*   On the liability side, the liability value will increase and the magnitude will be more than the increase in asset because of the longer liability maturity.
*   Magenta Rock will have a difficult time in funding the guaranteed minimum return on the fixed annuity business. In addition, it will face liquidity challenge in funding the withdrawal from the annuity business

c(ii)
Magenta Rock could use stochastic modelling to measure interest risk rate. Stochastic modelling makes use of ESG to generate a random process of financial variables in concern, interest rates in our case. It allows a wide range of scenarios to be modelled, providing a distribution of possible loss of Magenta Rock. This could better reflect the interest rate tail risk due to extreme market conditions.

5. **Continued**

(d)   During your review of ALCO's practices, you notice that credit risk was overlooked when analyzing future asset cash flows.

Describe how Magenta Rock could incorporate its credit risk exposure into the projected asset cash flows.

Commentary on Question:
Candidates scored reasonably well on 5(d)

A credit migration model can be used to model probability of default, exposure, and loss given default to incorporate credit risk exposure into the projected asset cash flows.

(e)   You suggest to the ALCO that the volatility of the liability cash flows could be minimized using risk transfer methods or derivatives.

Explain how each of the following methods could be used to minimize the volatility of Magenta Rock's liability cash flows:

- Reinsurance
- Interest rate swap

Commentary on Question:
Part 5(e) was fairly easy for candidates and many received full credit.

Using stop loss reinsurance for Magenta Rock's life product would transfer claims liability to a reinsurer after a specified threshold. This would put a cap on Magenta Rock's claims liability, resulting in less volatile liability cash flows.

Magenta Rock could use an interest rate swap to receive fixed interest rate payments in exchange for variable rates. This will help stabilize the annuity interest rate needs, resulting in less volatility in liability cash flows.

6.   **Learning Objectives:**
1.   The candidate will understand the ERM framework and process and be able to apply them to organizations.

5.   The candidate will understand the approaches for managing risks and how an entity makes decisions about appropriate techniques.

Learning Outcomes:
(1a)   Recommend an appropriate framework for an organization's enterprise risk management and an acceptable governance structure.

(1c)   Demonstrate how to articulate an organization's risk appetite, desired risk profile, quantified risk tolerances, risk philosophy and risk objectives.

(5c)   Demonstrate the use of controls for retained and residual risks.

Sources:
ERM-119-14: Aggregation of risks and Allocation of Capital (Sections 4-7 Excluding 6.3) (LO 1)

ERM-143-20: Internal Controls Toolkit, Doxey, Ch. 1, pp. 11-17 & 27-35 (LO 5)

ERM-702-12: IAA Note on ERM for Capital and Solvency Purposes in the Insurance Industry, Pages 9–38 (LO 1)

Risk Appetite: Linkage with Strategic Planning Report (LO 1)

Exploring The Black Box: Working with inherent and residual risk (LO 5)

Commentary on Question:
Overall candidates did not perform well on this question. Parts (b) and (c) respectively scored the fewest grading points.

Solution:
(a)   Big Ben is committed to maintaining a strong capital base to support the risk associated with its business.

Describe one residual risk and one inherent risk that apply to Big Ben. Justify your response.

Commentary on Question:
Residual risk is the risk that remains after taking into account all mitigating actions that can reasonably be taken in order to manage the risk. Liquidity risk is a residual risk that Big Ben still faces even after implementation of an ALM strategy.

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6.   Continued

Inherent risk is a function of exposure and impact, not considering any mitigation factors. Regulatory risk is an inherent risk that Big Ben faces regardless of the mitigation actions they put in place.

(b)   Big Ben is implementing the Model Governance framework.

   (i)   Define the three major types of internal controls.

   (ii)   Provide an example of an internal control used by Big Ben in the Model Governance framework, for each type you identified in part (i).

Commentary on Question:
Many candidates did not correctly identify the three major types of controls for part (i). Candidates also did not focus their examples for part (ii) to the Model Governance framework as requested in the question.

Part (i):
The three major types of internal controls include:
1. **Preventive** controls are designed to keep errors from occurring.
2. **Detective** controls are designed to identify errors that may have already occurred.
3. **Corrective** controls are designed to correct errors that have identified (or detected).

Part (ii):
1. Preventive example: Big Ben required that high risk models be validated on a strict 3-year rotation schedule through a centralized model validation group.
2. Detective example: Big Ben requires that models that are not subject to validation be reviewed by an independent analyst (e.g. someone who was not the model developer) who is familiar with the model's topic and purpose.
3. Corrective example: The formal model validation exercise will require a report with a pass or fail grade, regardless of the findings. If the model fails, a remediation plan will need to be created by the model developer and executed in a timely manner.

6. **Continued**

(c)   Big Ben is formalizing its risk appetite framework.

   (i)   Describe the three increasingly detailed levels of a risk appetite framework.

   (ii)   Provide two examples of risk appetite that Big Ben already utilizes or is considering. Justify your response.

   (iii)   Provide two examples of risk tolerances that Big Ben already utilizes or is considering. Justify your response.

Commentary on Question:
Many candidates were unable to identify the detailed levels of the risk appetite framework in part (i) even though 2 of the 3 levels were given in parts (ii) and (iii). For examples provided in parts (ii) and (iii), the candidate was given credit for justification as either a risk appetite or risk tolerance, but not both.

Part (i):
1. **Enterprise Risk Tolerance** – the aggregate amount of risk the company is willing to take.
2. **Risk Appetite** – enterprise risk tolerance needs to be allocated to a risk appetite for specific risk categories and business activities.
3. **Risk Limit** – the most granular level used for business operation. A risk limit translates enterprise risk tolerance and risk appetite for each category into risk-monitoring measures.

Part (ii):
Example 1: Big Ben uses internal models to determine its required economic capital based on VaR. The quantile used for the VaR calculation is 99.5% over a one-year horizon.

Example 2: Big Ben requires that each line of business maintain an Internal Capital Adequacy Ratio of 140%.

Part (iii):
Example 1: To manage liquidity risk, the contractual maturities of assets and liabilities are monitored, and the duration mismatch is not allowed to exceed a specific tolerance.

Example 2: To manage interest rate risk, Big Ben will monitor the sensitivity of assets and liabilities to changes in interest rates. The Board wants to be able to withstand a 200bp parallel shift in the yield curve.

7.　**Learning Objectives:**
3.　　　The candidate will understand the concepts of risk modeling and be able to evaluate and understand the importance of risk models.

4.　　　The candidate will understand how the risks faced by an entity can be quantified and the use of metrics to measure risk.

5.　　　The candidate will understand the approaches for managing risks and how an entity makes decisions about appropriate techniques.

Learning Outcomes:
(3c)　Evaluate and select appropriate copulas as part of the process of modelling multivariate risks.

(3e)　Demonstrate the importance of the tails of distributions, tail correlations, and low frequency / high severity events, and the use of extreme value theory to analyze these situations.

(4a)　Determine risk exposures using common risk measures (e.g., VaR and TVaR) and compare the properties and limitations of such measures.

(4b)　Analyze quantitative financial and non-financial data using appropriate statistical methods to assist in quantifying risk.

(4c)　Analyze risks that are not easily quantifiable, such as liquidity, operational, and environmental risks.

(5i)　Choose appropriate techniques to measure, model and manage various financial and non-financial risks faced by an organization.

Sources:
ERM-618-20: Chapter 9, section 6 of Managing Investment Portfolios (INV)

ERM-619-20: Correlation: Pitfalls and Alternatives (INV)

Commentary on Question:
The goal of this question is to test a candidate's understanding of different risk measures and proper use of correlations. It serves the purpose.

7. Continued

As long as a candidate understands the questions, knows concepts and recalls the formulas, the candidate should be able to receive full credits, the judgements required are straight forward.

Candidate did well on this question overall. Some credits were lost due to
- *Misunderstanding or misinterpretation of the question or what returns given.*
- *Using wrong formulas, especially the formulas for denominator in ratios.*

Solution:

(a)   Calculate the following based on the observed returns. Show your work.

   (i)   Sharpe ratio, assuming a risk-free rate of 2%.

   (ii)   Sortino ratio, assuming a minimum acceptable return of 0%.

   (iii) Return over maximum drawdown.

Commentary on Question:
Most candidates did well on this part, errors were found due to misinterpretation of the returns, or formulas in ratio calculations.

(a) (i)

$$\text{Mean portfolio return} = \frac{10\% - 7\% - 5\% + 21\%}{4} = 4.75\%$$

Standard deviation of portfolio returns

$$= \sqrt{\frac{(10\% - 4.75\%)^2 + (-7\% - 4.75\%)^2 + (-5\% - 4.75\%)^2 + (21\% - 4.75\%)^2}{4 - 1}}$$

$$= 13.23\%$$

$$\text{Sharpe ratio} = \frac{\text{Mean portfolio return} - \text{Risk free rate}}{\text{Standard deviation of portfolio returns}}$$

$$\text{Sharpe ratio} = \frac{4.75\% - 2\%}{13.23\%} = 0.21$$

7. Continued

(a) (ii)

$$\text{Sortino ratio} = \frac{\text{Mean portfolio return} - \text{Minimum acceptable return}}{\text{Downside deviation}}$$

From part (i), the mean portfolio return is 4.75%.
Downside deviation is the standard deviation of the portfolio returns using data points only below 0%, i.e. -7% and -5%.

$$\text{Mean portfolio return of points below MAR} = \frac{-7\% - 5\%}{2} = -6\%$$

$$\text{Downside deviation} = \sqrt{\frac{\left(-7\% - (-6\%)\right)^2 + \left(-5\% - (-6\%)\right)^2}{2-1}} = 1.41\%$$

$$\text{Sortino ratio} = \frac{4.75\% - 0\%}{1.41\%} = 3.36$$

(a) (iii)
The stock's low point following its initial peak at end of Q1 occurred at end of Q3. Therefore, maximum drawdown can be determined by linking the Q2 and Q3 returns as follows:

$$\text{Maximum drawdown} = -[(1 - 0.07)(1 - 0.05) - 1] = 11.65\%$$

$$\text{Return over maximum drawdown} = \frac{\text{Mean portfolio return}}{\text{Maximum drawdown}} = \frac{4.75\%}{11.65\%} = 0.41$$

(b) Justify the appropriate performance metric from part (a) to use in each of the following scenarios.

(i) You believe that observed loss patterns over longer periods of time are the best proxy for actual exposure.

(ii) You believe that risk-adjusted return should not be negatively affected in periods of outsized positive performance.

(iii) You believe that an investment strategy that is expected to earn the risk-free rate should have a risk-adjusted return of exactly zero.

Commentary on Question:
Most candidates did well on this part, as long as they understand and are able to interpret the results, they pick the right ratios. Those who did not understand the ratios tend to answer them wrong, which is rare.

(i)
Return over maximum drawdown
RoMAD is the average return in a given year that a portfolio generates, expressed as a percentage of the drawdown. This is a way of expressing the risk for investors who believe the observed loss pattern is the best proxy for exposure.

(ii)
Sortino ratio
The numerator of the Sortino ratio is the return in excess of the minimum acceptable return (MAR). The denominator is the downside deviation using MAR as target return. This express the view that portfolio managers should not be penalized for volatility from outsized performance.

(iii)
Sharpe ratio
The Sharpe ratio is the mean return earned in excess of the risk-free rate. By subtracting risk-free rate from the mean return, we can isolate the performance associated with risk-taking activities.

(c)   You assign your analyst to compare SmartCo's historical returns against those of NGN, another company currently in the portfolio. Your analyst makes the following statements:

I.   If I can fit marginal distributions to SmartCo's and NGN's returns individually, I will have the information I need to jointly model SmartCo's and NGN's returns in a Monte Carlo simulation.

II. Under the simplifying assumption that SmartCo and NGN returns jointly follow an elliptical distribution, a correlation coefficient of zero will imply that the returns are independent.

Critique your analyst's statements.

Commentary on Question:
This is a straight froward question, as long as the candidates have the knowledge, they should be able to apply it correctly and receive full credits.

Statement I is incorrect. Marginal distributions and correlation will generally not determine a unique joint distribution. More specifically, Statement I is only correct in the case of elliptical distributions, but incorrect in the case of non-elliptical distributions.

Statement II is incorrect. The only case where a correlation coefficient of zero implies independence is in the specific case of the multivariate normal distribution. This relationship does not hold for other elliptical distributions (e.g. multivariate t-distribution).

7. **Continued**

(d)   Assess whether correlation is an appropriate measure of dependency for the two companies' returns. Justify your response based on analysis of the graph.

Commentary on Question:
Almost all candidates can identify the tail dependency and make the right call.

Correlation is not an appropriate measure of dependency in this case:

- The graph exhibits high tail dependence, as low returns for SmartCo tend to coincide with low returns for NextGen (i.e. lots of clustering near the line y = x for small values)

- For distributions that exhibit tail dependence, correlation is generally not an appropriate dependency measure as it would fail to capture the degree of tail dependence.

8. Learning Objectives:

4. The candidate will understand how the risks faced by an entity can be quantified and the use of metrics to measure risk.

5. The candidate will understand the approaches for managing risks and how an entity makes decisions about appropriate techniques.

Learning Outcomes:

(4b) Analyze quantitative financial and non-financial data using appropriate statistical methods to assist in quantifying risk.

(5d) Demonstrate how derivatives, synthetic securities, and financial contracting may be used to reduce risk within a static or dynamic hedging program.

Sources:

ERM-617-19: Options, Futures and Other Derivatives, Chapter 17; ERM-617-19: Options, Futures and Other Derivatives, Chapter 29

Commentary on Question:

This question tested candidates' understanding of delta hedging to mitigate the risk of writing a put option. Candidates generally did very well on the Black-Scholes calculations, and most got full credit for part (g) by recommending gamma hedging and/or more frequent rebalancing to improve hedge performance. On parts (e) and (f), almost no one answered the question as asked – most responses addressed breakage between the put option and the hedge whereas the question asked for the cost of the hedge (ie. gain/loss on the shares of the security underlying the put). Candidates received partial credit if their response demonstrated that they understood the mechanics of delta hedging by shorting shares at initiation and covering them in the future.

Solution:

(a) Calculate the number of put option contracts required. Show your work.

Number of options = (Beta * portfolio value) / (Index * multiplier) = 1.2 * 1,000,000 / (4000 * 100) = 3

(b) Calculate the strike price of the options. Show your work.

Required return = 950,000 / 1,000,000 – 1 = -5%
Under CAPM, Return = Risk-free rate + Beta * (Market return – Risk-free rate)
-5% = 2% + 1.2 * (Strike price / 4000 – 1 – 2%)
Strike price = 3847

8. Continued

(c)   Calculate the total cost of the put options. Show your work.

Pricing Formulas

By replacing S_0 by $S_0 e^{-qT}$ in the Black–Scholes–Merton formulas, equations (15.20) and (15.21), we obtain the price, c, of a European call and the price, p, of a European put on a stock paying a dividend yield at rate q as

$$c = S_0 e^{-qT} N(d_1) - K e^{-rT} N(d_2) \qquad (17.4)$$

$$p = K e^{-rT} N(-d_2) - S_0 e^{-qT} N(-d_1) \qquad (17.5)$$

Since

$$\ln \frac{S_0 e^{-qT}}{K} = \ln \frac{S_0}{K} - qT$$

it follows that d_1 and d_2 are given by

$$d_1 = \frac{\ln(S_0/K) + (r - q + \sigma^2/2)T}{\sigma\sqrt{T}}$$

$$d_2 = \frac{\ln(S_0/K) + (r - q - \sigma^2/2)T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T}$$

d1 = (ln(4000/3847) + (2% - 0% + (20%^2)/2) * 1)) / (20%*sqrt(1) = .3954
d2 = d1 – 20%*sqrt(1) = .1954
N(-d1) = .3463
N(-d2) = .4225
Put price per unit = (3847 * exp(-2%*1)) * .4225 – (4000 * exp(0*1)) * .3463 = 208.09
Put price in total = 208.09 * 3 * 1000 – 62,427

(d)   Calculate the delta on a single put option at time 0. Show your work.

Delta = -N(-d1) = -.3463

(e)   Describe how the cost arises from a delta hedge.

1. There is an initial gain from short-selling shares
2. There is an interest gain on the proceeds of (1)
3. There is a cost at time 1 of buying back the shares

(f)   Calculate the cost of the delta hedge after one year. Show your work.

Short .3463 shares per put to be hedged. Cost: -.3463 * 4000 * 3 = -4155.12
Interest on short position: 2% * -4155.12 = -83.10
Buy back (cover) shares at time 1. Proceeds: .3463 * 4400 * 3 = 4570.63
Total cost = 332.41 (a gain to the bank)

8. **Continued**

(g)   Recommend a method to improve the performance of the hedge. Justify your answer.

I recommend more frequent rebalancing. This will reduce Gamma exposure by more closely matching the put option Delta as it changes over time.

<A recommendation to gamma hedge using options was a common response, and also received full credit.>