SOCIETY OF ACTUARIES

Advanced Topics in General Insurance

Exam GIADV

Date: Friday, November 4, 2022

INSTRUCTIONS TO CANDIDATES

General Instructions

1. This examination has 8 questions numbered 1 through 8 with a total of 40 points.

   The points for each question are indicated at the beginning of the question.

2. While every attempt is made to avoid defective questions, sometimes they do occur. If you believe a question is defective, the supervisor or proctor cannot give you any guidance beyond the instructions provided in this document.

Written-Answer Instructions

1. Each question part or subpart should be answered either in the Word document or the Excel file as directed. Graders will only look at work in the indicated file.

   a) In the Word document, answers should be entered in the box marked ANSWER. The box will expand as lines of text are added. There is no need to use special characters or subscripts (though they may be used). For example, β_1 can be typed as beta_1 and σ^2 can be typed as sigma^2.

   b) Calculations should be done in Excel and entered as formulas. Performing calculations on scratch paper or with a calculator and then entering the answer in the cell will not earn full credit. Formatting of cells or rounding is not required for credit. Rows can be inserted to the answer input area as required to provide space for your answer.

   c) Individual exams may provide additional directions that apply throughout the exam or to individual items.

2. The answer should be confined to the question as set.

3. Prior to uploading your Word and Excel files, each file should be saved and renamed with your five-digit candidate number in the filename.

4. The Word and Excel files that contain your answers must be uploaded before the five-minute upload period expires.

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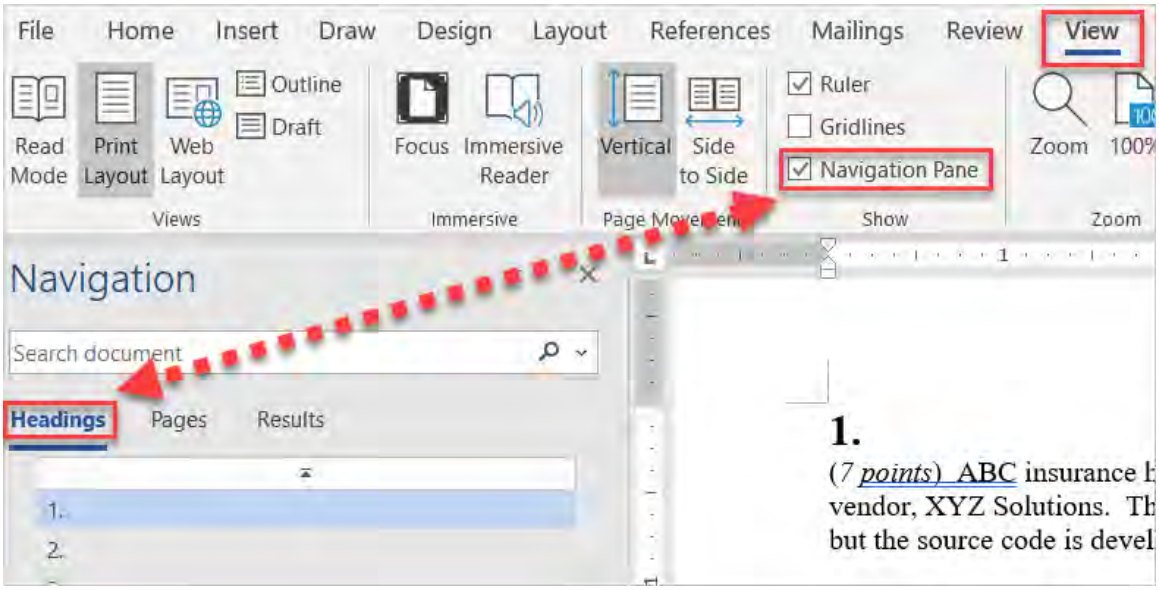
Navigation Instructions

Open the Navigation Pane to jump to questions.

Press Ctrl+F, or click View > Navigation Pane:



1.

(*4 points*) You are using exposure rating to price a workers compensation treaty. The treaty covers the layer 500,000 excess of 250,000. You are given the following:

State	Expected Loss Ratio
X	55%
Y	65%

State	Hazard Group	Standard Premium
X	C	200,000
X	D	180,000
Y	C	170,000
Y	D	230,000

You assume that the NCCI excess loss factor (ELF) curves can be approximated by an inverse power curve of the form $ELF_L = a\ L^{-b}$.

NCCI ELF Curve Estimation		
Parameter	**Hazard Group C**	**Hazard Group D**
a	8,000	1,200
b	1.00	0.75

(a) (*2.5 points*) Calculate the loss cost rate for the treaty.

Provide the response for this part in the Excel spreadsheet.

Casualty per occurrence excess treaties are often separated into three categories.

(b) (*1 point*) Describe the characteristics of the following categories:

(i) Working Layer

(ii) Exposed Excess

(iii) Clash Cover

Provide the response for this part in the Excel spreadsheet.

(c) (*0.5 points*) Compare the use of experience rating in pricing treaties in the three categories.

Provide the response for this part in the Excel spreadsheet.

2.

(*4 points*) You are determining the underwriting profit margin (UPM) using the Capital Asset Pricing Model (CAPM).

You are given the following assumptions about an insurer's line of business:

- The risk-free rate is 2%.
- The expected return on the market portfolio is 8%.
- The underwriting beta is 0.2.
- The tax rate on underwriting income is 25%.
- The ratio of premium to owner's equity is 2 to 1.

The line of business has three distinct payment pattern groups:

Group	Percentage of the Insurer's Business	Average Time Between Receipt of Premium and Payment of Losses and Expenses
1	50%	0.8 years
2	30%	1.1 years
3	20%	1.5 years

The investment portfolio backing the line of business has the following characteristics:

Asset	Percentage of Total Assets	Tax Rate
Tax exempt bonds	15%	0%
Corporate dividend income stocks	35%	10%
Taxable bonds	50%	25%

(a) (*1 point*) Describe two methods for obtaining the underwriting beta.

ANSWER:

(b) (*0.5 points*) Calculate the funds generating coefficient estimate, *k*.

Provide the response for this part in the Excel spreadsheet.

(c) (*1 point*) Calculate the UPM ignoring taxes.

Provide the response for this part in the Excel spreadsheet.

(d) (*1.5 points*) Calculate the UPM accounting for taxes.

Provide the response for this part in the Excel spreadsheet.

3.

(*5 points*) You are given the following data extracted from a triangle of cumulative paid losses:

Accident Year	From (months)	To (months)	Increment	Diagonal Age	Accident Year Total
2018	0	12	2,500	48	5,000
2018	12	24	1,800	48	5,000
2018	24	36	500	48	5,000
2018	36	48	200	48	5,000
2019	0	12	4,100	36	7,000
2019	12	24	2,000	36	7,000
2019	24	36	900	36	7,000
2020	0	12	4,600	24	6,800
2020	12	24	2,200	24	6,800
2021	0	12	5,300	12	5,300

You apply Clark's stochastic reserving model using the LDF method and an exponential distribution with cumulative distribution function $G(x) = 1 - e^{-x/\theta}$ where x is in months.

The maximum likelihood estimate (MLE) of θ is 8.1547.

(a) (*1 point*) Calculate the MLEs of *ULT* for each accident year.

Provide the response for this part in the Excel spreadsheet.

(b) (*0.5 points*) Calculate the value of the loglikelihood function at its maximum.

Provide the response for this part in the Excel spreadsheet.

(c) (*0.5 points*) Estimate σ^2, the scale factor.

Provide the response for this part in the Excel spreadsheet.

The contribution for accident year 2021 to the estimate of σ^2 is zero.

(d) (*1 point*) Explain why an accident year with a single incremental value will always contribute zero to the estimate.

Provide the response for this part in the Excel spreadsheet.

3. Continued

(e) (*0.5 points*) Estimate the process variance of the reserve for accident year 2020.

Provide the response for this part in the Excel spreadsheet.

(f) (*1.5 points*) Estimate the parameter variance of the reserve for accident year 2020.

Provide the response for this part in the Excel spreadsheet.

4.

(*9 points*) You are interested in determining the variability of unpaid claim estimates. The triangle of paid claims data you are working with, by accident year (AY) and development year, is presented below. It is assumed that all claims are fully developed after seven years.

Mack's method of estimating reserve variability is applied to this triangle.

	Development Year								
AY	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**Reserve**	**Standard Error**
1	2,089	7,443	12,056	14,891	17,770	19,852	20,727	0	0
2	3,096	9,116	11,930	16,779	18,552	20,232		892	198
3	2,382	8,774	11,820	16,451	18,064			2,750	477
4	1,899	7,537	12,697	16,974				5,130	1,251
5	1,538	6,670	9,658					7,221	1,592
6	1,928	7,197						11,305	2,931
7	2,579							21,382	4,670
Overall								48,679	6,651
f_k	3.61406	1.47094	1.34208	1.13019	1.10357	1.04408			
σ_i^2	494.976	221.738	72.280	43.233	6.426	0.955			

(a) (*2.5 points*) Demonstrate that the reserve and standard error for accident year 5 have been correctly calculated.

Provide the response for this part in the Excel spreadsheet.

(b) (*0.5 points*) Estimate the coefficient of variation of the unpaid claims for each of accident years 2-7 and overall.

Provide the response for this part in the Excel spreadsheet.

(c) (*1.5 points*) Estimate the upper 90% confidence limit of the overall unpaid claims using a lognormal distribution. (Note: The 90th percentile of a standard normal distribution is 1.28.)

Provide the response for this part in the Excel spreadsheet.

(d) (*1.5 points*) Allocate the overall amount from part (c) to accident years 2-7 in such a way to reach the same level of confidence for each accident year. (Note: Using Excel's Goal Seek function is an acceptable approach.)

Provide the response for this part in the Excel spreadsheet.

4. Continued

(e) (*0.5 points*) Explain the empirical approach to establishing confidence limits as described by Mack.

Provide the response for this part in the Excel spreadsheet.

(f) (*0.5 points*) Explain why this empirical approach to establishing confidence limits does not seem to be reasonable.

Provide the response for this part in the Excel spreadsheet.

(g) (*1 point*) Create a scatter plot to check the assumption that the expected losses at age 2 are proportional to the losses at age 1.

Provide the response for this part in the Excel spreadsheet.

(h) (*1 point*) Interpret the scatter plot in part (g) with regard to determining whether the assumption is correct. If it is not, recommend an alternative model.

Provide the response for this part in the Excel spreadsheet.

5.

(*4 points*) In "An Application of Game Theory: Property Catastrophe Risk Load," Donald Mango (Mango) used Kreps' formula to produce a risk load multiplier for the Marginal Surplus method applied to a portfolio.

You are given the following information:

- Return on marginal surplus is 20%
- Standard normal multiplier is 2.0 (corresponding to a cumulative non-exceedance probability of 97.725%)

(a) (*1 point*) Calculate the risk load multiplier using Kreps' formula.

ANSWER:

(b) (*0.5 points*) Describe how Mango converted this multiplier for use in the Marginal Variance method to ensure that the two methods have the same total risk load for the portfolio.

ANSWER:

Mango refers to portfolio variance as a *super-additive characteristic function*.

(c) (*1 point*) Explain what is meant by this reference.

ANSWER:

(d) (*1 point*) Compare the Shapley Value under a variance-based method to the Marginal Variance for calculating a risk load when adding a new account to an existing portfolio.

ANSWER:

(e) (*0.5 points*) Explain why Mango did not pursue the use of a Shapley Value under a standard deviation-based method.

ANSWER:

6.

(*5 points*) You are calculating a risk margin for insurance liabilities using the methodology set out in "A Framework for Assessing Risk Margins."

A balanced scorecard approach is used to determine the internal systemic risk coefficients of variation (CoVs) by line of business. You are given the following information:

Amount	Motor	Home
Outstanding Claim Liabilities (CL)	5,351,000	2,486,000
Premium Liabilities (PL)	4,215,000	3,993,000

	Potential Risk Indicators for Internal Systemic Risk	Score for	
		Motor CL	Home CL
1	Best predictors have been identified	5	4
2	Knowledge of past processes affecting predictors	3	3
3	Range of results produced by models	6	2
4	Number and importance of subjective adjustments to factors	4	3
5	Extent, timeliness, consistency and reliability of information from business	3	5
6	Value of predictors used	5	3

Internal Systemic Risk Correlation Matrix				
	Motor CL	Motor PL	Home CL	Home PL
Motor CL	100%	60%	35%	30%
Motor PL	60%	100%	15%	40%
Home CL	35%	15%	100%	50%
Home PL	30%	40%	50%	100%

Score from Balanced Scorecard Assessment	Motor CoV	Home CoV
1.0 to 1.5	19.0%	18.0%
1.5 to 2.0	15.0%	13.0%
2.0 to 2.5	12.5%	10.5%
2.5 to 3.0	10.5%	8.5%
3.0 to 3.5	9.0%	7.5%
3.5 to 4.0	8.0%	6.5%
4.0 to 4.5	7.5%	6.0%
4.5 to 5.0	7.0%	5.5%
5.0 to 6.0	6.5%	5.0%

6. Continued

- Risk indicator weights are equal within each source of internal systemic risk.
- Risk indicator weights for each source of internal systemic risk are:
  - 25% for specification error,
  - 55% for parameter selection error, and
  - 20% for data error.

(a) (*2 points*) Calculate the internal systemic risk CoV for each of the following:

(i) Motor CL

(ii) Home CL

(iii) Total CL

Provide the response for this part in the Excel spreadsheet.

The internal systemic risk CoVs for PL are as follows:

Motor PL	Home PL	Total PL
6.50%	6.00%	5.24%

(b) (*2 points*) Calculate the internal systemic risk CoV for total insurance liabilities, both lines combined.

Provide the response for this part in the Excel spreadsheet.

You are given the following additional information:

CoVs for total insurance liabilities	
Independent Risk	5.25%
External Systemic Risk	7.60%

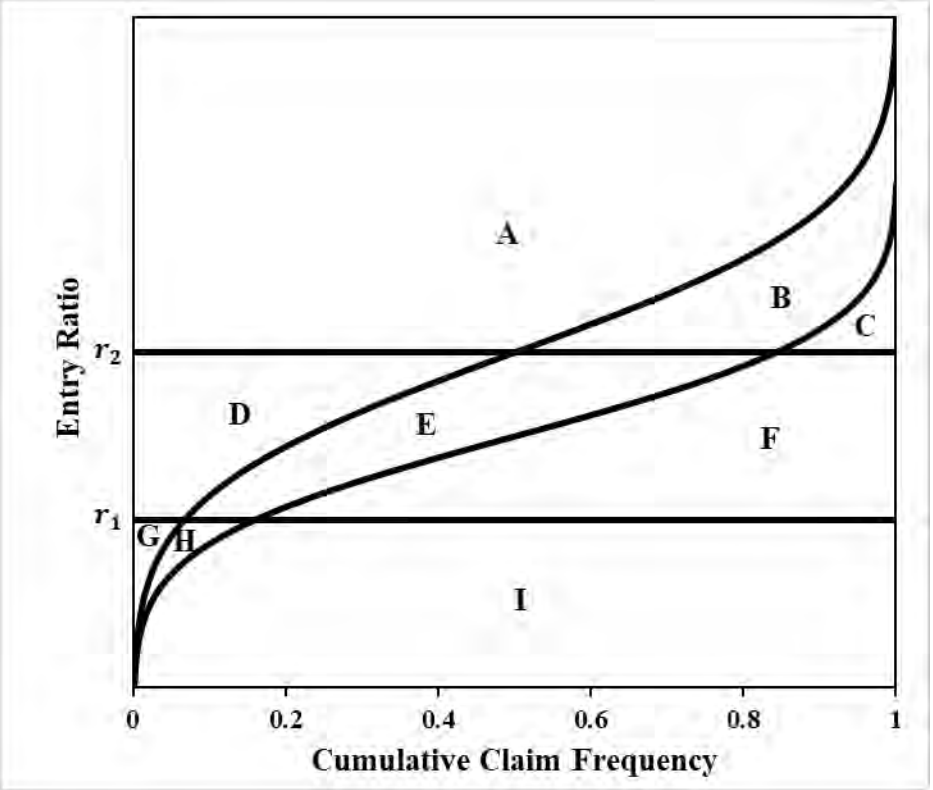
- The correlation between internal systemic risk and external systemic risk is assumed to be 20%.
- The total insurance liabilities are assumed to be normally distributed.
- The z-value of the 75th percentile of the Standard Normal distribution is 0.674.

(c) (*1 point*) Calculate the risk margin for the total insurance liabilities at the 75% adequacy level.

Provide the response for this part in the Excel spreadsheet.

7.

(*4 points*) You are given the following graph of the loss distribution for a risk with a retrospective rating plan, both with and without a per accident loss limitation:



Express the following quantities using the labels for the nine areas on the graph:

(i) 1

ANSWER:

(ii) k, the loss elimination ratio

ANSWER:

(iii) $\psi(r_1)$, the Table M savings

ANSWER:

7. Continued

(iv) $\psi^*(r_1)$, the Table L savings

ANSWER:

(v) $\phi(r_2)$, the Table M charge

ANSWER:

(vi) $\phi^*(r_2)$, the Table L charge

ANSWER:

(vii) $E\{L\}/E$, the ratio of the expected limited loss to the expected unlimited loss without a per accident loss limitation

ANSWER:

(viii) $E\{L^*\}/E$, the ratio of the expected limited loss to the expected unlimited loss with a per accident loss limitation

ANSWER:

8.

(*5 points*)

(a) (*0.5 points*) Describe what is meant by "collective risk model."

Provide the response for this part in the Excel spreadsheet.

You are assessing your company's reinsurance program for the management of catastrophe risk. The annual number of catastrophe losses has a Poisson distribution with mean 1.5. The loss size distribution for catastrophes is as follows:

Loss Size (billions)	Probability
1	0.6
2	0.3
3	0.1

Loss sizes are independent of one another and independent of the number of losses.

(b) (*1.5 points*) Calculate the mean and coefficient of variation for the aggregate losses.

Provide the response for this part in the Excel spreadsheet.

(c) (*3 points*) Complete the following aggregate loss probability table:

Aggregate Losses (billions)	Probability
0	
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	
13	
14	
15	

Provide the response for this part in the Excel spreadsheet.

****END OF EXAMINATION****